Event-triggered Control

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Where innovation starts

Acknowledgements

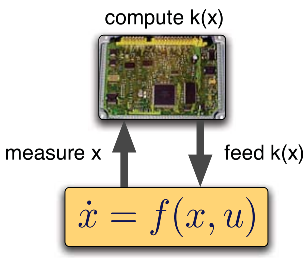
- Paulo Tabuada & Kalle Johansson (EECI course)

- Duarte Antunes
- Niek Borgers
- Tijs Donkers
- Tom Gommans

Introduction

Real-time control

- Feedback control is typically implemented on microprocessors using periodic time-triggered execution [1]



- Most of existing control theory was developed by ignoring the real-time implementation details: $u(t) = k(x(t))$
- Object of analysis: $\dot{x} = f(x, k(x))$

[1] courtesy Paulo Tabuada

Introduction

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Real-time control

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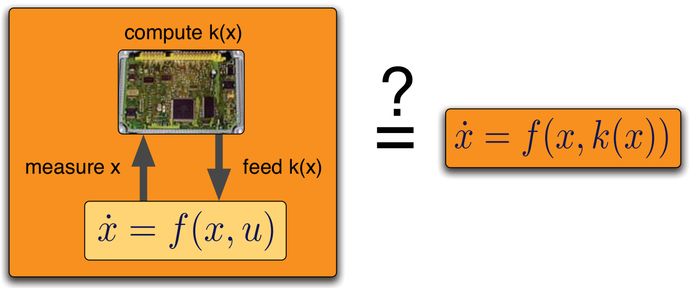

- Most of existing control theory was developed by ignoring the real-time implementation details: $u(t) = k(x(t))$
- Object of analysis: $\dot{x} = f(x, k(x))$

[1] courtesy Paulo Tabuada

/department of mechanical engineering

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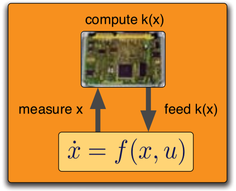
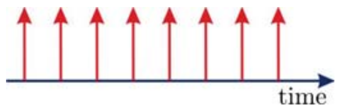
Introduction

Real-time control

- Feedback control is typically implemented on microprocessors using periodic time-triggered execution [1]



$\overset{?}{=}$ $\dot{x} = f(x, k(x))$



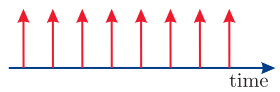
- Most of existing control theory was developed by ignoring the real-time implementation details.
- Assumption: Computation and communication sufficiently fast
- Separation of concerns with periodic time-triggered paradigm:
  - Control engineers design controllers while ignoring implementations
  - Software engineers schedule tasks while ignoring their functionality

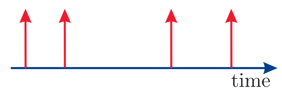
[1] courtesy Paulo Tabuada

Introduction

Periodic or Aperiodic: That's the question!

- Separation of concerns simplifies the design process, but results in inefficient usage of resources
- Paradigm shift: Periodic control $\longrightarrow$ Aperiodic control

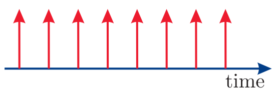


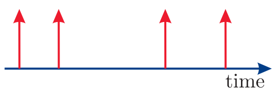


- Technological motivation:
  Resource-constrained large-scale cyber-physical systems
  - Computation time on embedded systems
  - Network utilisation in NCSs
  - Battery power in WCSs

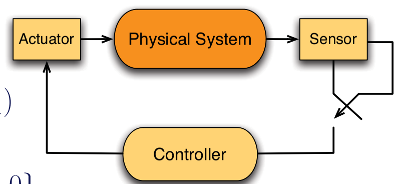
Introduction

Paradigm shift: Periodic control $\longrightarrow$ Aperiodic control





- Event-triggered control: reactive



$$u(t) \;=\; K(x(t_k)), \text{ when } t \in [t_k, t_{k+1})$$

$$t_{k+1} \;=\; \inf\{t > t_k \mid C(x(t), x(t_k)) \geqslant 0\}$$

- Example event-triggering condition

$$\|x(t) - x(t_k)\| \geqslant \sigma\|x(t)\| + \delta$$

Outline

First hour

- Basic setup for event-triggered control: State feedback case
- Two analysis frameworks
  - Perturbation approach
  - Hybrid system approach

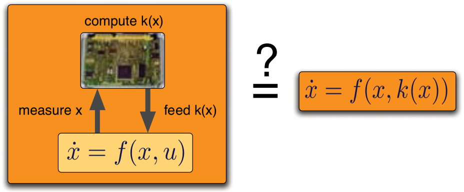
Second hour

- Output-feedback event-triggered control
- Robustness in event-triggered control
- Conclusions and outlook

Real-time control

- Linear system

$$\dot{x}(t) = Ax(t) + Bu(t)$$

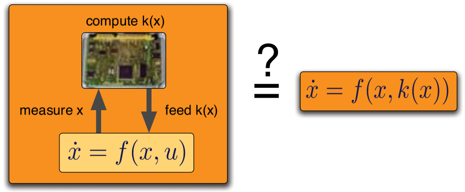


- State feedback $u(t) = Kx(t)$
- Ideal loop: $\dot{x}(t) = (A+BK)x(t)$
- Closed-loop stability: $A+BK$ Hurwitz matrix

Real-time control

- Linear system

$$\dot{x}(t) = Ax(t) + Bu(t)$$



- State feedback $u(t) = Kx(t)$
- Ideal loop: $\dot{x}(t) = (A+BK)x(t)$
- Sampled-data control with execution times t_k, $k \in \mathbb{N}$

$$u(t) = Kx(t_k), \quad t \in [t_k, t_{k+1})$$

- Aperiodic control : Inter-execution times $t_{k+1} - t_k$ varying
- Perturbation perspective: implementation-induced error

$$e(t) = x(t_k) - x(t) \text{ for } t \in [t_k, t_{k+1})$$

$$\dot{x}(t) = Ax(t) + BKx(t_k) = (A+BK)x(t) + BKe(t)$$

[1] Tabuada, TAC 2007

Event-triggered control

- Linear system

$$\dot{x}(t) = Ax(t) + Bu(t)$$

- Sampled-data control with execution times t_k, $k \in \mathbb{N}$

$$u(t) = Kx(t_k), \quad t \in [t_k, t_{k+1})$$

- Perturbation perspective:

$$\dot{x}(t) = Ax(t) + BKx(t_k) = (A+BK)x(t) + BKe(t)$$

- For ideal loop: Since $A+BK$ Hurwitz, quadratic Lyapunov function $V(x) = x^\top Px$ s.t.

$$\dot{V} \leqslant -c\|x\|^2$$

- For perturbed loop:

$$\dot{V} \leqslant -a^2\|x\|^2 + b^2\|e\|^2$$

Event-triggered control

- Perturbation perspective:

$$\dot{x}(t) = Ax(t) + BKx(t_k) = (A+BK)x(t) + BKe(t)$$

- Since $A+BK$ Hurwitz, quadratic Lyapunov function $V(x) = x^\top Px$ s.t.

$$\dot{V} \leqslant -a^2\|x(t)\|^2 + b^2\|e(t)\|^2$$

- Question: How to design event-trig. mech. determining t_k s.t. GES?

$$t_{k+1} = \inf\{t > t_k \mid C(x(t), e(t)) \geqslant 0\}$$

Event-triggered control

- Perturbation perspective:

$$\dot{x}(t) = Ax(t) + BKx(t_k) = (A+BK)x(t) + BKe(t)$$

- Since $A+BK$ Hurwitz, quadratic Lyapunov function $V(x) = x^\top Px$ s.t.

$$\dot{V} \leqslant -a^2\|x(t)\|^2 + b^2\|e(t)\|^2$$

- Question: How to design event-trig. mech. determining t_k s.t. GES?

$$t_{k+1} = \inf\{t > t_k \mid C(x(t), e(t)) \geqslant 0\}$$

- Crux: Guarantee $\|e(t)\| \leqslant \sigma a/b \cdot \|x(t)\|$ with $0 < \sigma < 1$ s.t.

$$\dot{V} \leqslant -a^2\|x(t)\|^2 + b^2\|e(t)\|^2 \leqslant -(1-\sigma^2)a^2\|x(t)\|^2$$

- Guarantee for Global Exponential Stability

$$t_{k+1} = \inf\{t > t_k \mid \| \underbrace{x(t_k) - x(t)}_{=e(t)} \| \geqslant \sigma a/b \cdot \|x(t)\|\}$$

Event-triggered control

- Summary of event-triggered setup:
  - Linear system

$$\dot{x}(t) = Ax(t) + Bu(t)$$

  - Execution times t_k, $k \in \mathbb{N}$

$$t_{k+1} = \inf\{t > t_k \mid \|x(t_k) - x(t)\| \geqslant \sigma a/b \cdot \|x(t)\|\}$$

  - Control law:

$$u(t) = Kx(t_k), \quad t \in [t_k, t_{k+1})$$

- Global exponential stability (GES)
- Question: Which important issue should we still verify?

Minimal inter-event times

- Summary of event-triggered setup:
  - Linear system

$$\dot{x}(t) = Ax(t) + Bu(t)$$

  - Execution times t_k, $k \in \mathbb{N}$

$$t_{k+1} = \inf\{t > t_k \mid \|x(t_k) - x(t)\| \geqslant \sigma a/b \cdot \|x(t)\|\}$$

  - Control law:

$$u(t) = Kx(t_k), \quad t \in [t_k, t_{k+1})$$

- Minimal inter-event time (MIET)

$$\inf\{t_{k+1} - t_k \mid k \in \mathbb{N}\}$$

- MIET should have a strictly positive lower bound!

Minimal inter-event times

- Inter-execution times:

$$t_{k+1} = \inf\{t > t_k \mid \| \underbrace{x(t_k) - x(t)}_{=e(t)} \| \geqslant \sigma a/b \cdot \|x(t)\|\}$$

- Note after event-time: $e(t_k) = 0$
- Dynamics state:

$$\dot{x}(t) = (A+BK)x(t) + BKe(t)$$

- Dynamics induced error $e(t) = x(t_k) - x(t)$ for $t \in [t_k, t_{k+1})$

$$\dot{e}(t) = -\dot{x}(t) = -(A+BK)x(t) - BKe(t)$$

- Inter-event time: time needed for $\|e(t)\|^2 - \sigma^2 a^2/b^2 \cdot \|x(t)\|^2$ to reach 0

Minimal inter-event times

- Note after event-time: $e(t_k) = 0$

$$\begin{aligned} \dot{x}(t) &= (A+BK)x(t) + BKe(t) \\ \dot{e}(t) &= -\dot{x}(t) = -(A+BK)x(t) - BKe(t) \end{aligned}$$

- Inter-event time: time needed for $\|e(t)\|^2 - \sigma^2 a^2/b^2 \cdot \|x(t)\|^2$ to reach 0

$$\Downarrow$$

- Take $\xi = (x, e)$ with dynamics

$$\dot{\xi}(t) = \Phi\xi(t) \text{ with } \Phi = \begin{pmatrix} A+BK & BK \\ -(A+BK) & -BK \end{pmatrix} \text{ and } \xi(0) = \begin{pmatrix} I \\ 0 \end{pmatrix} x_0$$

- Inter-event determined by time needed for $\xi^\top(t)Q\xi(t)$ to reach 0 with

$$Q = \begin{pmatrix} -\sigma^2 a^2/b^2 I & 0 \\ 0 & I \end{pmatrix}$$

$\longrightarrow$ MIET: $\inf_{x_0 \neq 0} \inf\{t > 0 \mid x_0^\top (I\ 0) e^{\Phi^\top t} Q e^{\Phi t} \begin{pmatrix} I \\ 0 \end{pmatrix} x_0 = 0\}$

Minimal inter-event times

$\longrightarrow$ MIET $\inf_{x_0 \neq 0} \inf\{t > 0 \mid x_0^\top (I\ 0) e^{\Phi^\top t} Q e^{\Phi t} \begin{pmatrix} I \\ 0 \end{pmatrix} x_0 = 0\}$

- This leads to [eliminating x_0]

$$\inf\{t > 0 \mid (I\ 0) e^{\Phi^\top t} Q e^{\Phi t} \begin{pmatrix} I \\ 0 \end{pmatrix} \succeq 0\}$$

- Proof of strictly positive lower bound on MIET (global)
- Computable by non-conservative (!) eigenvalue test

Summary

- Linear systems $\dot{x}(t) = Ax(t) + Bu(t)$
- Sampled-data control with execution times t_k, $k \in \mathbb{N}$

$$u(t) = Kx(t_k), \quad t \in [t_k, t_{k+1})$$

- Perturbation perspective:

$$\dot{x}(t) = Ax(t) + BKx(t_k) = (A+BK)x(t) + BKe(t)$$

- Since $A + BK$ Hurwitz, quadratic Lyapunov function $V(x) = x^\top Px$ s.t.

$$\dot{V} \leqslant -a^2\|x(t)\|^2 + b^2\|e(t)\|^2$$

- Event-triggering mechanism

$$t_{k+1} = \inf\{t > t_k \mid \| \underbrace{x(t_k) - x(t)}_{=e(t)} \| \geqslant \sigma a/b \cdot \|x(t)\|\}$$

- Guarantees:
  - Closed-loop stability
  - (Global) lower bound on minimal inter-event times

Illustrative Examples

Example 1: State feedback control

- Consider $\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$ and $u(t) = [1\ \ -4]x(t_k)$
- Example taken from [1]
- TTC: $t_k = k \cdot 0.025$
- ETC: $t_k = t \iff \|e(t)\| = 0.05\|x(t)\|$

[1] Tabuada, TAC '07

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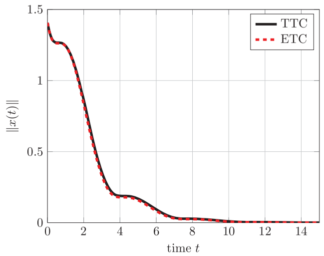

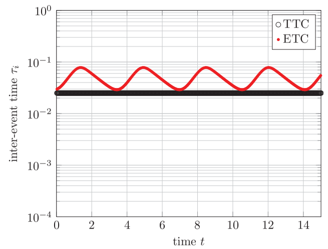

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Illustrative Examples

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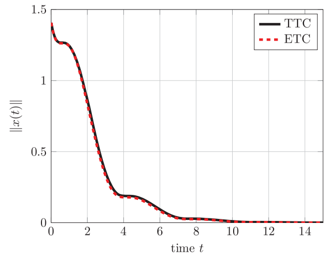

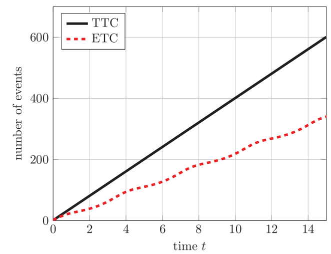

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Hybrid Systems Approach

- Analysis in perturbation approach based on
  - $\dot{x}(t) = Ax(t) + BKx(t_k) = (A + BK)x(t) + BKe(t)$
  - $\|e(t)\| \leqslant \sigma a/b \cdot \|x(t)\|$
- Ignores dynamics error $e(t) = x(t_k) - x(t)$ for $t \in [t_k, t_{k+1})$

$$\dot{e}(t) = -\dot{x}(t) = -(A + BK)x(t) - BKe(t)$$

- Hybrid system: $\xi = (x, e)$

$$\begin{aligned} \dot{\xi} &= \Phi\xi \quad \text{when } \xi^\top Q\xi \leqslant 0 \\ \xi^+ &= J\xi \quad \text{when } \xi^\top Q\xi \geqslant 0 \end{aligned}$$

$$\Phi = \begin{pmatrix} A + BK & BK \\ -(A + BK) & -BK \end{pmatrix} \quad Q = \begin{pmatrix} -\sigma^2 a^2/b^2 I & 0 \\ 0 & I \end{pmatrix} \quad J = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}$$

[1] Donkers, Heemels, CDC10 & TAC12
[2] Postoyan et al, CDC 11

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Hybrid Systems Approach

- Hybrid system: $\xi = (x, e)$

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- Stability analysis using hybrid tools [1]: $V(\xi) = \xi^\top P\xi$
  - $\frac{d}{dt}V(\xi) < 0$ when $\xi^\top Q\xi \leqslant 0$
  - $V(J\xi) \leqslant V(\xi)$ when $\xi^\top Q\xi \geqslant 0$
- Linear matrix inequalities: if there are $\alpha, \beta \geqslant 0$ s.t.
  - $\Phi^\top P + P\Phi - \alpha Q \prec 0$
  - $J^\top PJ - P + \beta Q \preceq 0$
- Guarantee for GES
- Never more conservative than perturbation approach [2]

[1] Goebel, Sanfelice, Teel, CSM09
[2] Donkers, Heemels, TAC12

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Illustrative Examples

Example 1: Comparison P and HS approach

- Consider $\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$ and $u(t) = \begin{bmatrix} 1 & -4 \end{bmatrix} x(t_k)$
- Example taken from [1]
- We look for largest $\tilde{\sigma}$ giving stability: $\|e\|^2 \leqslant \tilde{\sigma}\|x\|^2$ [2]

	$\tilde{\sigma}$	MIET
P: Results from [1]	0.0030	0.0318
P: By minimising the $\mathcal{L}_2$-gain	0.0273	0.0840
Hybrid System	0.0588	0.1136

- PS: via minimising $\mathcal{L}_2$-gain: minimise b/a to maximise a/b

$$\dot{V} \leqslant -a^2\|x(t)\|^2 + b^2\|e(t)\|^2 \quad \text{for } \dot{x} = (A+BK)x + BKe$$

- ETM:

$$t_{k+1} = \inf\{t > t_k \mid \|\underbrace{x(t_k) - x(t)}_{=e(t)}\| \geqslant \sigma a/b \cdot \|x(t)\|\}$$

[1] Tabuada, TAC '07 [2] Donkers, Heemels, CDC10 & TAC12

Nonlinear Systems

Do similar results hold for the nonlinear case?

- Nonlinear system $\dot{x} = f(x, u)$
- Sampled-data control with execution times t_k, $k \in \mathbb{N}$

$$u(t) = k(x(t_k)) = k(x(t) + e(t)), \quad t \in [t_k, t_{k+1})$$

- Perturbation perspective: $\dot{x} = f(x, k(x+e))$
- Suppose Input-to-State Stability (ISS) with ISS LF V

$$\frac{\partial V}{\partial x} f(x, k(x+e)) \leqslant -\alpha(\|x\|) + \beta(\|e\|)$$

- α, β class $\mathcal{K}$-functions: $\alpha(0) = 0$, continuous, strictly increasing
- Event-triggering mechanism: for some $0 < \sigma < 1$

$$-\alpha(\|x\|) + \beta(\|e\|) \leqslant -\sigma\alpha(\|x\|)$$

- $t_{k+1} = \inf\{t > t_k \mid \beta(\|e(t)\|) \geqslant -(1-\sigma)\alpha(\|x(t)\|)\}$
- Stability & lower bound on minimal inter-event times (Lipschitz)

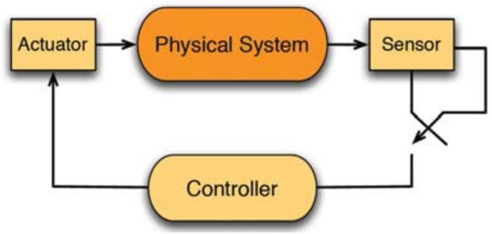
Event-triggered control

Complete design

- Emulation-based design procedure for K
- Using perturbation approach we can synthesize ETM $\|e\| \geqslant \sigma\|x\|$
- ETM (σ) can be pushed further using hybrid system approach
- Guaranteed GES
- Guaranteed positive and global lower bound on MIET

Are we done??

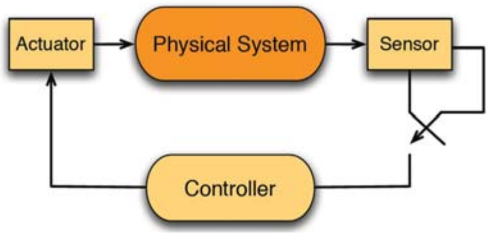
Output-based ETC



- What if full state x not available for feedback, but only output y?

Output-based ETC

Illustrative example



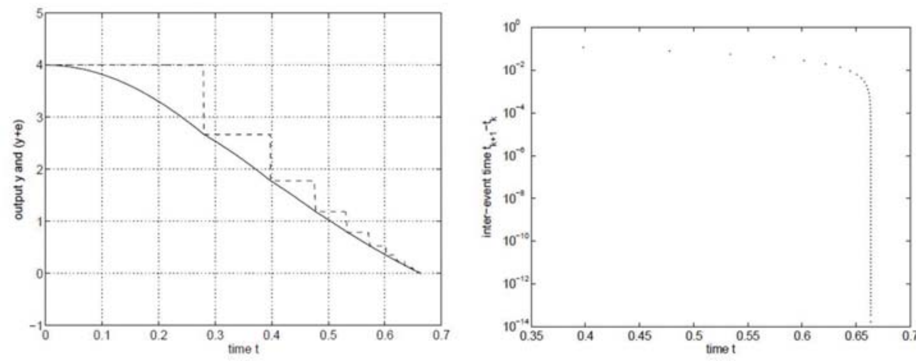
- Consider

$$\begin{cases} \dot{x}_p = \begin{bmatrix} 1 & -1 \\ 10 & -1 \end{bmatrix} x_p + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u \\ y = \begin{bmatrix} 1 & 0 \end{bmatrix} x_p \end{cases} \qquad u(t) = -2y(t_k)$$

- ETM: $\|y(t) - y(t_k)\|^2 > \sigma \|y(t)\|^2$
- Parameter: $\sigma = 0.5$

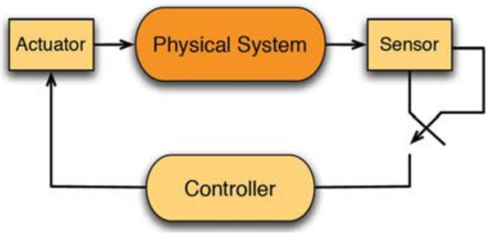
Output-based ETC

Illustrative example



- Minimal inter-event time (MIET) is zero!
- Zeno behavior: an infinite number of events in a finite length interval

Disturbances in ETC

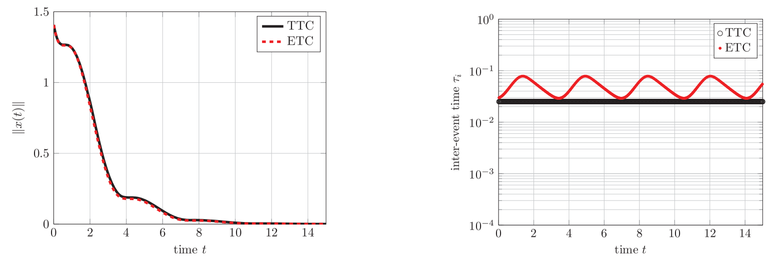


- What if small disturbances are present?

Disturbances in ETC

Illustrative example

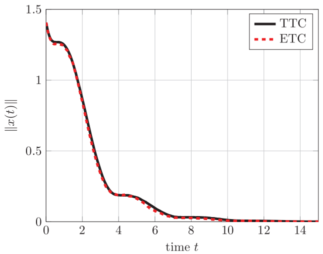
- Consider $\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$ and $u(t) = \begin{bmatrix} 1 & -4 \end{bmatrix} x(t_k)$
- TTC: $t_k = k \cdot 0.025$
- ETC: $t_k = t \iff \|e(t)\| = 0.05\|x(t)\|$

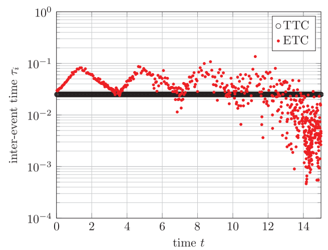


Disturbances in ETC

Illustrative example

- Consider $\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + w$ and $u(t) = \begin{bmatrix} 1 & -4 \end{bmatrix} x(t_k)$
- TTC: $t_k = k \cdot 0.025$
- ETC: $t_k = t \iff \|e(t)\| = 0.05\|x(t)\|$





What's next?

Issues in ETC

- Output-based ETC
- Robustness of MIET / resource utilization

Outline 2nd hour

- Solution I for output-based ETC:
  - Alternative event-triggering conditions (not relative)
- Event-separation properties for alternative ETC (robustness)
- Solution II for output-based ETC:
  - Time regularization: Periodic Event-Triggered Control (PETC)
- Again perturbation and hybrid system approaches
- Conclusions & Outlook

Output-based control

Output-based ETC

Possible remedies

- Solution I: Adopt alternative ETMs instead of $\|y - \hat{y}\|^2 > \sigma\|y\|^2$
  - Absolute: $\|y - \hat{y}\|^2 > \varepsilon$
  - Mixed: $\|y - \hat{y}\|^2 > \sigma\|y\|^2 + \varepsilon$
- Solution II: Time regularization
  - Enforce minimal inter-event time T [1,2]

$$t_{k+1} = \inf\{t > t_k + T \mid \|y - \hat{y}\|^2 > \sigma\|y\|^2\}$$

  - Events only at kh, $k \in \mathbb{N}$

$$t_{k+1} = \inf\{t > t_k \mid \|y - \hat{y}\|^2 > \sigma\|y\|^2 \wedge t = kh, \ k \in \mathbb{N}\}$$

  ▶ Periodic Event-Triggered Control (PETC)

[1] Tallapragada & Chopra. CDC12 & NECSYS12
[2] Heemels, Sandee, Van den Bosch, IJC08

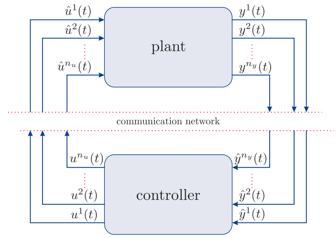
Output-based ETC using Mixed ETM

The Event-Triggered Control System

System Description

Objective:

- Setup an output-based event-triggering mechanism (ETM)
- Guaranteed MIET > 0
- Mixed ETM: $\|y^i - \hat{y}^i\| > \sigma_i \|y^i\| + \varepsilon_i$
- General setup: decentralized ETM



[Donkers, Heemels, TAC 2012]

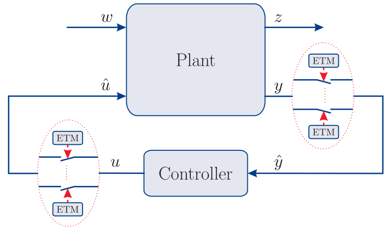
Overview for mixed ETMs

- Mixed ETM: $\|y^i - \hat{y}^i\| > \sigma_i \|y^i\| + \varepsilon_i$
- Minimal inter-event time strictly positive guaranteed (bound depends on $\|\bar{x}(0)\|$ and $\|w\|$)
- Ultimate boundedness: determined by σ_i
- Size ultimate bound: tuned by ε_i
- Two perspectives for LMI-based stability and $\mathcal{L}_\infty$-gain analysis
  - Hybrid system approach
  - Perturbation approach
  - HS less conservative, but harder work
- Tradeoff ultimate bound/$\mathcal{L}_\infty$-gain and number of events

[Donkers, Heemels, TAC 2012]

Illustrative Examples

Example 2: $\varepsilon_i = 0$ zero inter-event times!



- Consider

$$\begin{cases} \dot{x}_p = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} x_p + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \hat{u} \\ y = \begin{bmatrix} -1 & 4 \end{bmatrix} x_p \end{cases} \qquad \begin{cases} \dot{x}_c = \begin{bmatrix} 0 & 1 \\ 0 & -5 \end{bmatrix} x_c + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \hat{y} \\ u = \begin{bmatrix} 1 & -4 \end{bmatrix} x_c \end{cases}$$

- ETM: $\|\hat{y} - y\|^2 = \sigma_1 \|y\|^2 + \varepsilon_1$ and $\|\hat{u} - u\|^2 = \sigma_1 \|u\|^2 + \varepsilon_2$
- Parameters: $\sigma_1 = \sigma_2 = 10^{-3}$ and $\varepsilon_1 = \varepsilon_2 = 0$

Illustrative Examples

Example 2: $\varepsilon_i = 0$ zero inter-event times!

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$$\begin{cases} \dot{x}_p = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} x_p + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \hat{u} \\ y = \begin{bmatrix} -1 & 4 \end{bmatrix} x_p \end{cases} \qquad \begin{cases} \dot{x}_c = \begin{bmatrix} 0 & 1 \\ 0 & -5 \end{bmatrix} x_c + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \hat{y} \\ u = \begin{bmatrix} 1 & -4 \end{bmatrix} x_c \end{cases}$$

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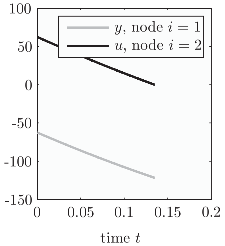
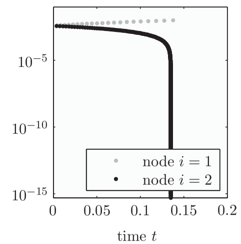




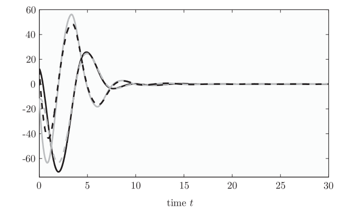
Illustrative Examples

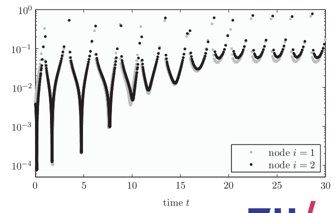
Example 2: Need for extending ETM including ε_i

- Consider

$$\begin{cases} \dot{x}_p = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} x_p + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \hat{u} \\ y = \begin{bmatrix} -1 & 4 \end{bmatrix} x_p \end{cases} \qquad \begin{cases} \dot{x}_c = \begin{bmatrix} 0 & 1 \\ 0 & -5 \end{bmatrix} x_c + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \hat{y} \\ u = \begin{bmatrix} 1 & -4 \end{bmatrix} x_c \end{cases}$$

- ETM: $\|\hat{y} - y\|^2 = \sigma_1 \|y\|^2 + \varepsilon_1$ and $\|\hat{u} - u\|^2 = \sigma_1 \|u\|^2 + \varepsilon_2$
- Parameters: $\sigma_1 = \sigma_2 = 10^{-3}$ and $\varepsilon_1 = \varepsilon_2 = 10^{-3}$





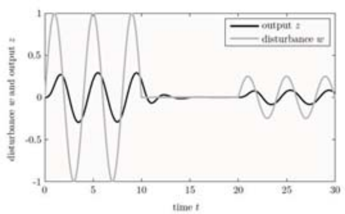
Illustrative Examples

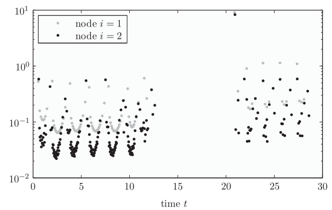
Example 3: What ETC is all about!

- Consider

$$\begin{cases} \dot{x}_p = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x_p + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \hat{u} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w \\ y = \begin{bmatrix} 1 & 0 \end{bmatrix} x_p \end{cases} \qquad \begin{cases} \dot{x}_c = \begin{bmatrix} -2 & 1 \\ -13 & -3 \end{bmatrix} x_c + \begin{bmatrix} -2 \\ -5 \end{bmatrix} \hat{y} \\ u = \begin{bmatrix} 5 & 2 \end{bmatrix} x_c \end{cases}$$

- Taking $\sigma_1 = \sigma_2 = 10^{-3}$ and $\varepsilon_1 = \varepsilon_2 = 10^{-3}$, yields $\mathcal{L}_\infty$-gain of 0.46 for $z = [1\ 0]x_p$





- Act when needed!

ETC under disturbances

Disturbances in ETC

Event-separation properties

- Consider $\dot{x} = Ax + Bu + w$ and $u(t) = Kx(t_k) = K(x(t) + e(t))$
- Inter-execution times:

$$t_{k+1} = \inf\{t > t_k \mid \|\underbrace{x(t_k) - x(t)}_{=e(t)}\| \geqslant \sigma\|x(t)\| + \varepsilon\}$$

→ MIET $\tau(x_0, w)$ dependent on x_0 and w: $\tau(x_0, w) = \inf_{k\in\mathbb{N}} (t_{k+1} - t_k)$

→ Different kinds of triggering conditions

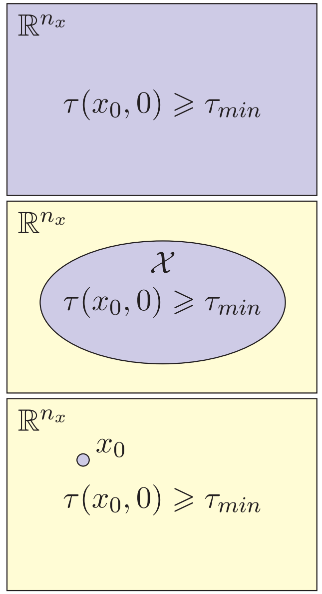
- Relative ETM: $\varepsilon = 0$ and $\sigma > 0$
- Absolute ETM: $\varepsilon > 0$ and $\sigma = 0$
- Mixed ETM: $\varepsilon > 0$ and $\sigma > 0$

Event-separation properties

Global: Lower bound $\tau_{min} > 0$ for the complete $\mathbb{R}^{n_x}$ in absence of disturbances

Semi-global: Lower bound $\tau_{min} > 0$ for any compact set $\mathcal{X} \subset \mathbb{R}^{n_x}$ in absence of disturbances

Local: Lower bound $\tau_{min} > 0$ for any single point $x_0 \in \mathbb{R}^{n_x}$ in absence of disturbances

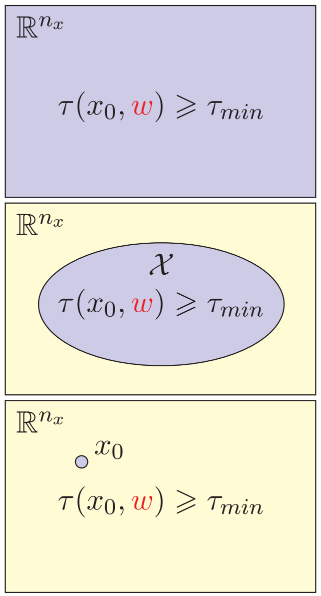


Event-separation properties

Robust Global: Lower bound $\tau_{min} > 0$ for the complete $\mathbb{R}^{n_x}$ for bounded disturbances $\|w\|_{\mathcal{L}_\infty} \leqslant \epsilon$

Robust Semi-global: Lower bound $\tau_{min} > 0$ for any compact set $\mathcal{X} \subset \mathbb{R}^{n_x}$ for bounded disturbances $\|w\|_{\mathcal{L}_\infty} \leqslant \epsilon$

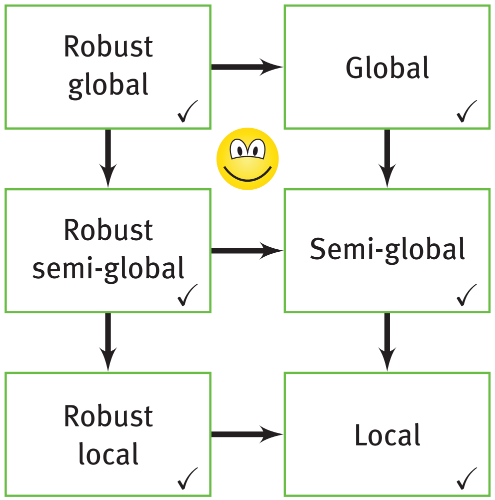
Robust Local: Lower bound $\tau_{min} > 0$ for any single point $x_0 \in \mathbb{R}^{n_x}$ for bounded disturbances $\|w\|_{\mathcal{L}_\infty} \leqslant \epsilon$



Overview of event-separation properties

Robust global ✓	Global
Robust semi-global	Semi-global
Robust local	Local

Overview of event-separation properties



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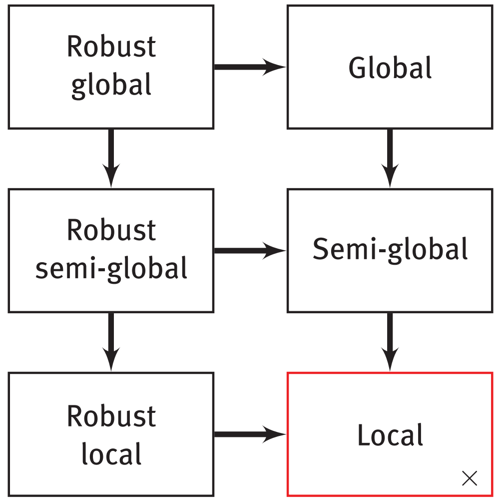
Overview of event-separation properties



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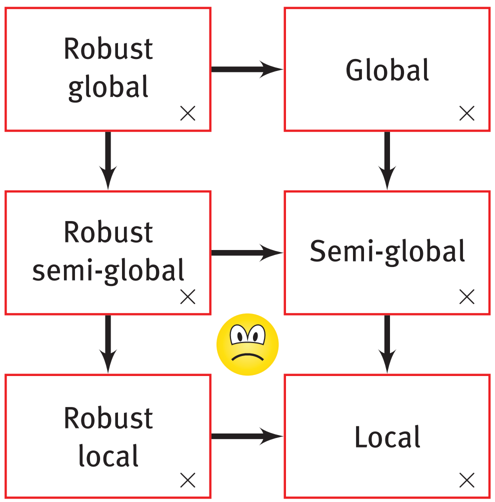

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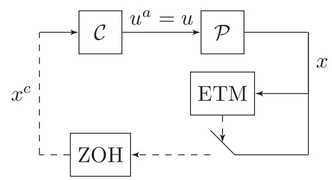
Example: Linear system



$$\mathcal{P} : \dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$\mathcal{C} : u = \begin{bmatrix} 1 & -4 \end{bmatrix} (x + e)$$

rel ETM $\quad t_i = t \iff \|e(t)\| = 0.05\|x(t)\|$

abs ETM $\quad t_i = t \iff \|e(t)\| = 0.001$

mix ETM $\quad t_i = t \iff \|e(t)\| = 0.05\|x(t)\| + 0.001$

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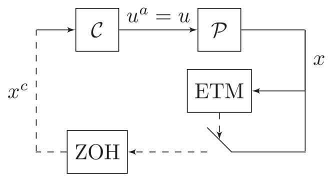
Example: Linear system



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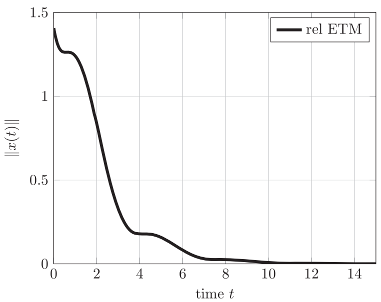
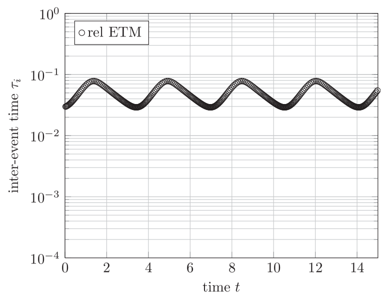
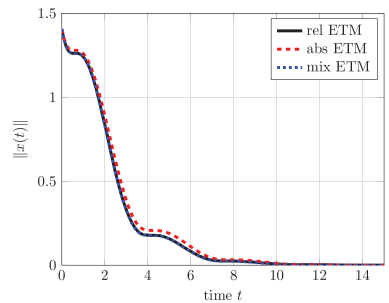
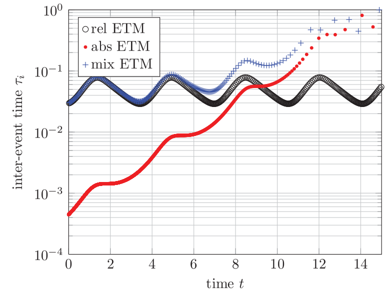




Example: Linear system

ETM	robust global	global	robust semi-global	semi-global	robust local	local
relative		✓		✓		✓
absolute		×		✓		✓
mixed		✓		✓		✓





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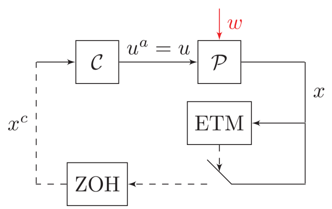

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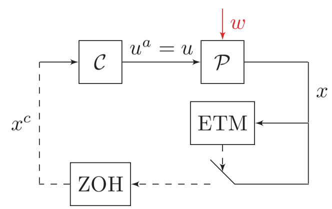

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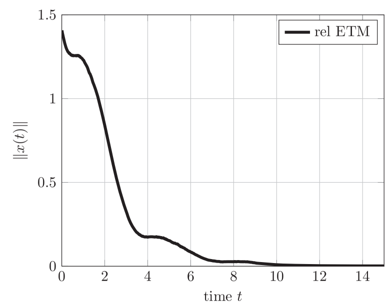
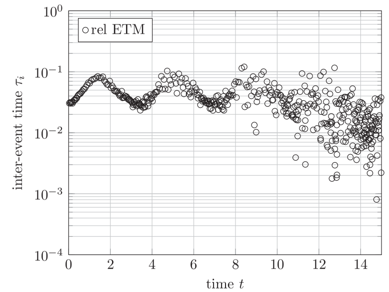
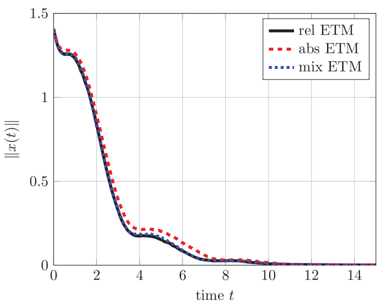
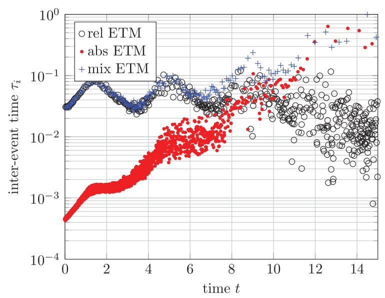




Example: Linear system

ETM	robust global	global	robust semi-global	semi-global	robust local	local
relative	×	✓	×	✓	×	✓
absolute	×	×	✓	✓	✓	✓
mixed	✓	✓	✓	✓	✓	✓





Summary

- Event-separation properties: local/semi-global/global and robustness of MIET/resource utilization

State-feedback case

ETM	robust global	global	robust semi-global	semi-global	robust local	local
relative	×	✓	×	✓	×	✓
absolute	×	×	✓	✓	✓	✓
mixed	✓	✓	✓	✓	✓	✓

Output-feedback case

ETM	robust global	global	robust semi-global	semi-global	robust local	local
relative	×	×	×	×	×	×
absolute	×	×	✓	✓	✓	✓
mixed	×	×	✓	✓	✓	✓

- Relative triggering fragile. Zero robustness
- Mixed or absolute effective (semi-global) but UB (no GAS)

[Borgers & Heemels, CDC 13 submitted]

Periodic Event-Triggered Control (PETC)

Periodic ETC

- Solution I: Adopt alternative ETMs instead of $\|y-\hat{y}\|^2 > \sigma\|y\|^2$
  - Mixed: $\|y-\hat{y}\| > \sigma\|y\| + \varepsilon$
- Solution II: Time regularization
  - Events only at kh, $k \in \mathbb{N}$

$$t_{k+1} = \inf\{t > t_k \mid \|y-\hat{y}\|^2 > \sigma\|y\|^2 \wedge t = kh,\ k \in \mathbb{N}\}$$

▶ Periodic Event-Triggered Control (PETC) [1,2]

[1] Heemels, Donkers & Teel, CDC/ECC'11 [2] Heemels, Donkers & Teel , TAC 13

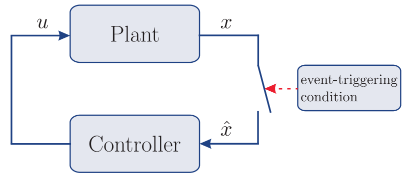
Periodic ETC (PETC)

- Paradigm shift: Periodic control $\longrightarrow$ Event-triggered control

- In contrast with CETC ....
- .... Sampling periodic, but ...
- .... Every sampling time it is decided to act or not.
- PETC vs. CETC: Implementation advantages!
  - Guaranteed (reasonable) minimal inter-event time
  - Only time-periodic verification of event-triggering conditions
  - More in line with time-sliced architectures

PETC

Description



$$\tfrac{\mathrm{d}}{\mathrm{d}t}x = A^p x + B^p u + B^w w$$

$$u(t) = K\hat{x}(t)$$

$$\hat{x}(t) = \begin{cases} x(t_k), & \text{when } \mathcal{C}(x(t_k), \hat{x}(t_k)) > 0 \\ \hat{x}(t_k), & \text{when } \mathcal{C}(x(t_k), \hat{x}(t_k)) \leqslant 0 \end{cases} \quad \text{for } t \in (t_k, t_{k+1}]$$

with $t_k = kh$ and $h > 0$ fixed sampling period

PETC

$$\hat{u}(t) = K\hat{x}(t),$$

$$\hat{x}(t) = \begin{cases} x(t_k), & \text{when } \mathcal{C}(x(t_k), \hat{x}(t_k)) > 0 \\ \hat{x}(t_k), & \text{when } \mathcal{C}(x(t_k), \hat{x}(t_k)) \leqslant 0 \end{cases}$$

- [1,2] $\|\hat{x}(t_k) - x(t_k)\| > \sigma\|x(t_k)\|$
- [3] $\|K\hat{x}(t_k) - Kx(t_k)\| > \sigma\|Kx(t_k)\|$
- [4,5] $(Ax(t_k) + BK\hat{x}(t_k))^\top P(Ax(t_k) + BK\hat{x}(t_k)) > \beta\, x^\top(t_k)Px(t_k)$

$$\mathcal{C}(\xi(t_k)) = \xi^\top(t_k)Q\xi(t_k) > 0,$$

where $\xi := (x, \hat{x}) \in \mathbb{R}^{n_\xi}$

[1] Tabuada, TAC '07
[2] Wang & Lemmon, TAC '10
[5] Mazo et al, ECC'09
[3] Donkers & Heemels CDC'10,TAC'12
[4] Velasco et al, CDC'09

PETC

Hybrid system formulation

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix}\xi \\ \tau\end{bmatrix} = \begin{bmatrix}\bar{A}\xi + \bar{B}w \\ 1\end{bmatrix}, \quad \text{when } \tau \in [0, h],$$

$$\begin{bmatrix}\xi^+ \\ \tau^+\end{bmatrix} = \begin{cases} \begin{bmatrix}J_1\xi \\ 0\end{bmatrix}, & \text{when } \xi^\top Q\xi > 0,\ \tau = h \\ \begin{bmatrix}J_2\xi \\ 0\end{bmatrix}, & \text{when } \xi^\top Q\xi \leqslant 0,\ \tau = h \end{cases}$$

with $\xi := (x, \hat{x}) \in \mathbb{R}^{n_\xi}$ and

$$\bar{A} := \begin{bmatrix}A^p & B^pK \\ 0 & 0\end{bmatrix}, \quad \bar{B} := \begin{bmatrix}B^w \\ 0\end{bmatrix}, \quad J_1 := \begin{bmatrix}I & 0 \\ I & 0\end{bmatrix}, \quad J_2 := \begin{bmatrix}I & 0 \\ 0 & I\end{bmatrix}$$

PETC

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix}\xi\\ \tau\end{bmatrix} = \begin{bmatrix}\bar{A}\xi + \bar{B}w\\ 1\end{bmatrix}, \text{ when } \tau \in [0,h],$$

$$\begin{bmatrix}\xi^+\\ \tau^+\end{bmatrix} = \begin{cases}\begin{bmatrix}J_1\xi\\ 0\end{bmatrix}, & \text{when } \xi^\top Q\xi > 0,\ \tau = h\\ \begin{bmatrix}J_2\xi\\ 0\end{bmatrix}, & \text{when } \xi^\top Q\xi \leqslant 0,\ \tau = h\end{cases}$$

Problem formulation:

- Globally exponentially stable $(w = 0)$: $\|\xi(t)\| \leqslant ce^{-\rho t}\|\xi(0)\|$
- $\mathcal{L}_2$ gain smaller than or equal to γ with $z = \bar{C}\xi + \bar{D}w$

$$\sqrt{\int_0^\infty \|z(t)\|^2 dt} \leqslant \gamma\sqrt{\int_0^\infty \|w(t)\|^2 dt}$$

PETC

Problem formulation:

- Globally exponentially stable $\|\xi(t)\| \leqslant ce^{-\rho t}\|\xi(0)\|$
- $\mathcal{L}_2$ gain smaller than or equal to γ with $z = \bar{C}\xi + \bar{D}w$

$$\sqrt{\int_0^\infty \|z(t)\|^2 dt} \leqslant \gamma\sqrt{\int_0^\infty \|w(t)\|^2 dt}$$

Three approaches:

(i) Discrete-time piecewise linear system (PWL) approach

(ii) Discrete-time Perturbation approach $\rightarrow$ Similar

(iii) Hybrid system approach

[1] Heemels, Donkers & Teel , TAC 13

Piecewise linear system approach

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix}\xi\\ \tau\end{bmatrix} = \begin{bmatrix}\bar{A}\xi + \bar{B}w\\ 1\end{bmatrix}, \text{ when } \tau \in [0,h],$$

$$\begin{bmatrix}\xi^+\\ \tau^+\end{bmatrix} = \begin{cases}\begin{bmatrix}J_1\xi\\ 0\end{bmatrix}, & \text{when } \xi^\top Q\xi > 0,\ \tau = h\\ \begin{bmatrix}J_2\xi\\ 0\end{bmatrix}, & \text{when } \xi^\top Q\xi \leqslant 0,\ \tau = h\end{cases}$$

- Discretize at $t_k = kh$ with $\xi_k := \xi(t_k)$ $(w = 0)$

$$\xi_{k+1} = \begin{cases}A_1\xi_k, & \text{when } \xi_k^\top Q\xi_k > 0,\\ A_2\xi_k, & \text{when } \xi_k^\top Q\xi_k \leqslant 0, \text{ where}\end{cases}$$

$$A_1 = e^{\bar{A}h}J_1 = \begin{bmatrix}A+BK & 0\\ I & 0\end{bmatrix}, \quad A_2 = e^{\bar{A}h}J_2 = \begin{bmatrix}A & BK\\ 0 & I\end{bmatrix},$$

$$A := e^{A^p h}, \quad B := \int_0^h e^{A^p s}B^p ds.$$

Piecewise linear system approach

$$\xi_{k+1} = \begin{cases}A_1\xi_k, & \text{when } \xi_k^\top Q\xi_k > 0,\\ A_2\xi_k, & \text{when } \xi_k^\top Q\xi_k \leqslant 0, \text{ where}\end{cases}$$

- Piecewise quadratic Lyapunov function:

$$V(\xi) = \begin{cases}\xi^\top P_1\xi, & \text{when } \xi^\top Q\xi > 0,\\ \xi^\top P_2\xi, & \text{when } \xi^\top Q\xi \leqslant 0,\end{cases}$$

Theorem The PETC system is GES, if there are matrices P_1, P_2 and scalars $\alpha_{ij} \geqslant 0$, $\beta_{ij} \geqslant 0$ and $\kappa_i \geqslant 0$, s.t.

$$P_i - A_i^\top P_j A_i + (-1)^i\alpha_{ij}Q + (-1)^j\beta_{ij}A_i^\top Q A_i \succ 0, \text{ for all } i,j \in \{1,2\},$$

and

$$P_i + (-1)^i\kappa_i Q \succ 0, \text{ for all } i \in \{1,2\}.$$

Hybrid system approach

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix}\xi\\ \tau\end{bmatrix} = \begin{bmatrix}\bar{A}\xi + \bar{B}w\\ 1\end{bmatrix}, \text{ when } \tau \in [0,h],$$

$$\begin{bmatrix}\xi^+\\ \tau^+\end{bmatrix} = \begin{cases}\begin{bmatrix}J_1\xi\\ 0\end{bmatrix}, & \text{when } \xi^\top Q\xi > 0,\ \tau = h\\ \begin{bmatrix}J_2\xi\\ 0\end{bmatrix}, & \text{when } \xi^\top Q\xi \leqslant 0,\ \tau = h\end{cases}$$

Main idea $\mathcal{L}_2$ gain analysis: $z = \bar{C}\xi + \bar{D}w$

- Timer-dependent quadratic Lyapunov function $V(\xi,\tau) = \xi^\top P(\tau)\xi$
- $\frac{\mathrm{d}}{\mathrm{d}t}V \leqslant -2\rho V - \gamma^{-2}z^\top z + w^\top w$
- During jumps decrease:

$$V(J_1\xi,0) \leqslant V(\xi,h), \text{ for all } \xi \text{ with } \xi^\top Q\xi > 0,$$
$$V(J_2\xi,0) \leqslant V(\xi,h), \text{ for all } \xi \text{ with } \xi^\top Q\xi \leqslant 0$$

Hybrid system approach

Main idea $\mathcal{L}_2$ gain analysis $V(\xi,\tau) = \xi^\top P(\tau)\xi$

- $\frac{\mathrm{d}}{\mathrm{d}t}V \leqslant -2\rho V - \gamma^{-2}z^\top z + w^\top w$
- During jumps decrease:

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Nice idea ... but how?

Hybrid system approach

Main idea $\mathcal{L}_2$ gain analysis $V(\xi,\tau) = \xi^\top P(\tau)\xi$

- $\frac{\mathrm{d}}{\mathrm{d}t}V \leqslant -2\rho V - \gamma^{-2}z^\top z + w^\top w$
- During jumps decrease:

$$V(J_1\xi,0) \leqslant V(\xi,h), \text{ for all } \xi \text{ with } \xi^\top Q\xi > 0,$$

Nice idea ... but how?

- Riccati differential equation ($M = (\gamma^{-2}\bar{D}^\top\bar{D} - I)^{-1}$)

$$\frac{\mathrm{d}}{\mathrm{d}\tau}P = -\bar{A}^\top P - P\bar{A} - 2\rho P - \gamma^{-2}\bar{C}^\top\bar{C} - (P\bar{B} + \gamma^{-2}\bar{C}^\top\bar{D})M(\bar{B}^\top P + \gamma^{-2}\bar{D}^\top\bar{C})$$

- Hamiltonian $H := \begin{bmatrix}\bar{A} + \rho I + \gamma^{-2}\bar{B}M\bar{D}^\top\bar{C} & \bar{B}M\bar{B}^\top\\ -\bar{C}^\top L\bar{C} & -(\bar{A} + \rho I + \gamma^{-2}\bar{B}M\bar{D}^\top\bar{C})^\top\end{bmatrix}$
- $F(\tau) := e^{-H\tau} = \begin{bmatrix}F_{11}(\tau) & F_{12}(\tau)\\ F_{21}(\tau) & F_{22}(\tau)\end{bmatrix}$
- $P_0 = (F_{21}(h) + F_{22}(h)P_h)\big(F_{11}(h) + F_{12}(h)P_h\big)^{-1}$

Hybrid system approach

Theorem: Suppose that there exist matrix $P_h \succ 0$, and scalars $\mu_i \geqslant 0$, such that for $i \in \{1,2\}$

$$\begin{bmatrix}P_h + (-1)^i\mu_i Q & J_i^\top\bar{F}_{11}^{-\top}P_h\bar{S} & J_i^\top(\bar{F}_{11}^{-\top}P_h\bar{F}_{11}^{-1} + \bar{F}_{21}\bar{F}_{11}^{-1})\\ \star & I - \bar{S}^\top P_h\bar{S} & 0\\ \star & \star & \bar{F}_{11}^{-\top}P_h\bar{F}_{11}^{-1} + \bar{F}_{21}\bar{F}_{11}^{-1}\end{bmatrix} \succ 0$$

Then, the PETC system is GES with convergence rate ρ (when $w = 0$) and has an $\mathcal{L}_2$-gain smaller than or equal to γ.

Comparison of the approaches

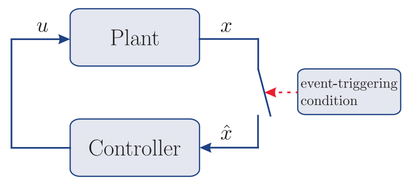
(i) Piecewise linear system approach
(ii) Perturbation approach (discrete-time linear system)
(iii) Hybrid system approach (intersample behavior)

- All LMI-based and thus efficient
- (ii): Simple: $\mathcal{H}_\infty$-norm calculations for GES
- (i): Least conservative for GES
- (iii): $\mathcal{L}_2$ gain analysis

[1] Heemels, Donkers & Teel , TAC 13

Numerical example

$$\frac{\mathrm{d}}{\mathrm{d}t}x = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$



$$u(t) = K\hat{x}(t)$$

$K = [1 \ -4]$ and $t_k = kh$ with $h = 0.05$. For $t \in (t_k, t_{k+1}]$

$$\hat{x}(t) = \begin{cases} x(t_k), & \text{when } \|\hat{x}(t_k) - x(t_k)\| > \sigma\|x(t_k)\| \\ \hat{x}(t_k), & \text{when } \|\hat{x}(t_k) - x(t_k)\| \leqslant \sigma\|x(t_k)\| \end{cases}$$

Numerical example

$$\hat{x}(t) = \begin{cases} x(t_k), & \text{when } \|\hat{x}(t_k) - x(t_k)\| > \sigma\|x(t_k)\| \\ \hat{x}(t_k), & \text{when } \|\hat{x}(t_k) - x(t_k)\| \leqslant \sigma\|x(t_k)\| \end{cases}$$

Maximal σ for which GES guaranteed by different approaches:

(i) $\sigma_{\text{pert}} := 0.1728$ ($\mathcal{H}_\infty = 1/0.1728$)
(ii) $\sigma_{\text{PWL}} = \sigma_{\text{HS}} = 0.2425$

As expected, $\sigma_{\text{pert}} \leqslant \sigma_{\text{PWL}}$ and $\sigma_{\text{HS}} \leqslant \sigma_{\text{PWL}}$

Minimal inter-event times:

- $2h = 0.1$ in former cases and $3h = 0.15$ in latter case!

Numerical example

$$\frac{\mathrm{d}}{\mathrm{d}t}x = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$u(t) = K\hat{x}(t)$$

$K = [1 \ -4]$ and $t_k = kh$ with $h = 0.05$. For $t \in (t_k, t_{k+1}]$

$$\hat{x}(t) = \begin{cases} x(t_k), & \text{when } \|K\hat{x}(t_k) - Kx(t_k)\| > \sigma\|Kx(t_k)\| \\ \hat{x}(t_k), & \text{when } \|K\hat{x}(t_k) - Kx(t_k)\| \leqslant \sigma\|Kx(t_k)\| \end{cases}$$

- Note: Could also be output-based example $u = \hat{y}$ and $y = Kx$

Numerical example

$$\frac{\mathrm{d}}{\mathrm{d}t}x = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$u(t) = K\hat{x}(t)$$

$K = [1 \;\; -4]$ and $t_k = kh$ with $h = 0.05$. For $t \in (t_k, t_{k+1}]$

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Maximal σ for which GES guaranteed by different approaches:

(i) $\sigma_{\text{PWL}} = 0.2550$

(ii) $\sigma_{\text{pert}} = 0.2506$

(iii) $\sigma_{\text{HS}} = 0.2532$

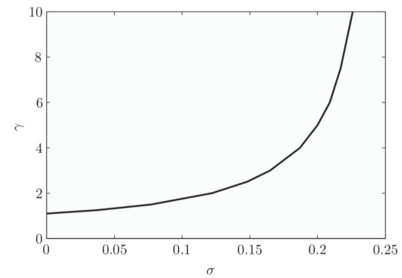
As expected, $\sigma_{\text{pert}} \leqslant \sigma_{\text{PWL}}$ and $\sigma_{\text{HS}} \leqslant \sigma_{\text{PWL}}$

Numerical example

$$\frac{\mathrm{d}}{\mathrm{d}t}x = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} 1 \\ 0 \end{bmatrix} w, \qquad u(t) = K\hat{x}(t)$$

$$\hat{x}(t) = \begin{cases} x(t_k), & \text{when } \|K\hat{x}(t_k) - Kx(t_k)\| > \sigma\|Kx(t_k)\| \\ \hat{x}(t_k), & \text{when } \|K\hat{x}(t_k) - Kx(t_k)\| \leqslant \sigma\|Kx(t_k)\| \end{cases}$$

- $\mathcal{L}_2$ gain analysis $z = [0\ 1]x$



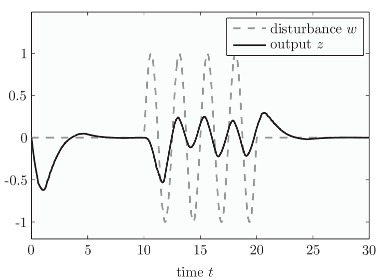
- $\sigma \downarrow 0$ then $\gamma \to \mathcal{L}_2$-gain of periodic s-d control (design)

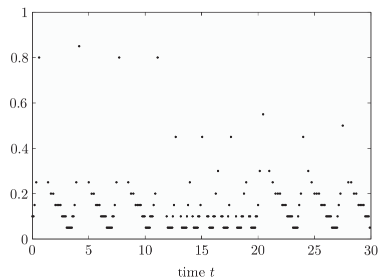
Numerical example

$$\frac{\mathrm{d}}{\mathrm{d}t}x = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} 1 \\ 0 \end{bmatrix} w, \qquad u(t) = K\hat{x}(t)$$

$$\hat{x}(t) = \begin{cases} x(t_k), & \text{when } \|K\hat{x}(t_k) - Kx(t_k)\| > \sigma\|Kx(t_k)\| \\ \hat{x}(t_k), & \text{when } \|K\hat{x}(t_k) - Kx(t_k)\| \leqslant \sigma\|Kx(t_k)\| \end{cases}$$

- $\mathcal{L}_2$ gain analysis $z = [0\ 1]x$





- $\sigma = 0.2$
- Inter-event times: 0.05 to 0.85 (17 times h)!!!!

Conclusions

- Part I: Event-triggered control based on full state feedback and relative triggering
  - Global asymptotic stability
  - Guaranteed global MIET
- Two analysis frameworks
  - Perturbation perspective (emulation-based design)
  - Hybrid systems perspective (less conservative)
- Output feedback and disturbances removed favorable features
- Part II: Output-based ETC and event-separation properties (ESP)
  - Absolute/Mixed triggering
    - ▶ UB/practical stability & robust (semi-global) ESP
  - Time regularization (Periodic Event-Triggered Control (PETC))
    - ▶ Time-regularization plus relative ETM: GAS, robust global ESP, but reduces to time-triggered loop under disturbances
  - Careful with robustness issues in resource utilization
  - Again perturbation and hybrid system approaches

Extensions & Outlook

- Linear system: $\dot{x} = Ax + Bu + w$
- Control law: $u = Kx(t_k) = K(x + e)$
- Execution times:

$$t_{k+1} = \inf\{t > t_k \mid \| \underbrace{x(t_k) - x(t)}_{=e(t)} \| \geqslant \sigma \|x(t)\| + \varepsilon\}$$

Extensions & Outlook

- Linear system: $\dot{x} = Ax + Bu + w$
- Control law: $u = Kx(t_k) = K(x + e)$
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$$t_{k+1} = \inf\{t > t_k \mid \|x(t_k) - x(t)\| \geqslant \sigma \|x(t)\| + \varepsilon\}$$

- Decentralized event-triggered control [1-6]

$$t_{k+1} = \inf\{t > t_k \mid \|y_i(t_k) - y_i(t)\| \geqslant \sigma_i \|y_i(t)\| + \varepsilon_i\}$$

[1] Mazo Jr. and Tabuada, TAC 11
[2] Persis et al, Automatica 2013
[3] Garcia, Antsaklis, ACC 2012
[4] Wang, Lemmon, TAC 11
[5] Donkers, Heemels, TAC 2012
[6] Heemels, Donkers, TAC 13, Automatica 13

Extensions & Outlook

- Linear system: $\dot{x} = Ax + Bu + w$
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$$t_{k+1} = \inf\{t > t_k \mid \|x(t_k) - x(t)\| \geqslant \sigma \|x(t)\| + \varepsilon\}$$

- Model-based event-triggered control [1,2,3,4,5]
  - "I know what you know principle "
  - $t_{k+1} = \inf\{t > t_k \mid \|x_{\text{pred}}(t) - x(t)\| \geqslant \sigma \|x(t)\| + \varepsilon\}$ with

$$\dot{x}_{\text{pred}}(t) = (A + BK)x_{\text{pred}}(t) \text{ with } x_{\text{pred}}(t_k) = x(t_k)$$

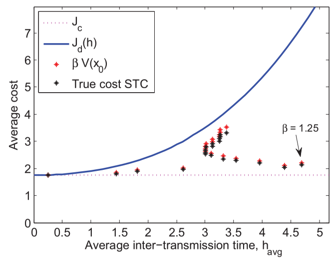
[1] Yook, Tilbury et al, TCST02
[2] Lunze, Lehmann, Automatica10
[3] Garcia, Antsaklis, ACC12
[4] Heemels, Donkers, Automatica13
[5] Bernardini, Bemporad, Automatica12

Extensions & Outlook

- Linear system: $\dot{x} = Ax + Bu + w$
- Control law: $u = Kx(t_k) = K(x + e)$
- Execution times:

$$t_{k+1} = \inf\{t > t_k \mid \|x(t_k) - x(t)\| \geqslant \sigma \|x(t)\| + \varepsilon\}$$

- Outperforming periodic time-triggered control [1,2,3]



[1] Astrom, Bernhardsson, IFAC 99, CDC02
[2] Barradas-Berglind et al, NMPC12
[3] Antunes, Heemels, Tabuada, CDC12

Extensions & Outlook

- Event-triggered and self-triggered control appealing research area
- System theory far from mature
- Many interesting problems open
- Good area for young researchers!
- Homepage

http://www.dct.tue.nl/heemels

Literature

- Lectures based on:
  - D.P. Borgers and W.P.M.H. Heemels, *Event-separation properties of event-triggered control systems*, IEEE Conference on Decision and Control 2013, submitted.
  - M.C.F. Donkers and W.P.M.H. Heemels, *Output-Based Event-Triggered Control with Guaranteed L_∞-gain and Improved and Decentralised Event-Triggering*, IEEE Transactions on Automatic Control, 57(6), p. 1362-1376, 2012.
  - W.P.M.H. Heemels, M.C.F. Donkers, and A.R. Teel, Periodic Event-Triggered Control for Linear Systems, IEEE Transactions on Automatic Control , 58(4), p. 847-861, 2013.
  - P. Tabuada, *Event-triggered real-time scheduling of stabilizing control tasks*, IEEE Trans. Autom. Control, vol. 52, no. 9, pp. 1680-1685, Sep. 2007.

Literature

- Mentioned extensions:
  - D. Antunes, W.P.M.H. Heemels, and Paulo Tabuada, *Dynamic Programming Formulation of Periodic Event-Triggered Control: Performance Guarantees and Co-Design*, 51st IEEE Conference on Decision and Control 2012, Hawaii, USA, p. 7212-7217.
  - J.D.J. Barradas Berglind, T.M.P. Gommans, and W.P.M.H. Heemels, *Self-triggered MPC for constrained linear systems and quadratic costs*, IFAC Conference on Nonlinear Model Predictive Control 2012, Noordwijkerhout, Netherlands, p. 342-348.
  - M.C.F. Donkers, P. Tabuada, and W.P.M.H. Heemels, "Minimum Attention Control for Linear Systems: A Linear Programming Approach", Discrete Event Dynamic Systems Theory and Applications , special issue "Event-Based Control and Optimization," 2012.
  - W. Heemels and M. Donkers, *Model-based periodic event-triggered control for linear systems*, Automatica, 49(3), March 2013, p. 698-711, 2013.

Literature

- Earlier works:
  - W.P.M.H. Heemels, J.H. Sandee, P.P.J. van den Bosch, "Analysis of event-driven controllers for linear systems", International Journal of Control, 81(4), pp. 571-590 (2008)
  - W.P.M.H. Heemels, R.J.A. Gorter, A. van Zijl, P.P.J. v.d. Bosch, S. Weiland, W.H.A. Hendrix, M.R. Vonder, Asynchronous measurement and control: a case study on motor synchronisation, Control Engineering Practice, 7(12), 1467-1482, (1999)
- Tutorial
  - W.P.M.H. Heemels, K.H. Johansson, and P. Tabuada, "An introduction to event-triggered and self-triggered control," 51st IEEE Conference on Decision and Control 2012, Hawaii, USA, p. 3270-3285

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