Resource-aware control

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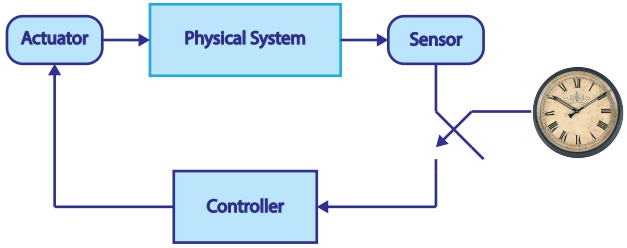
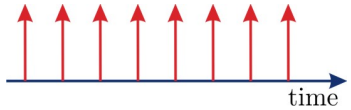
Where innovation starts

Introduction

Standard digital control loop

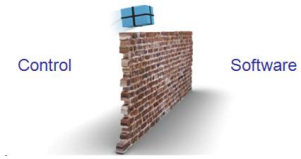


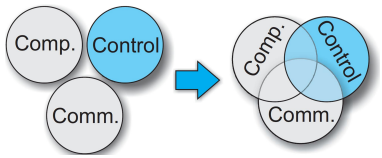


⟶ All control tasks executed periodically and triggered by time

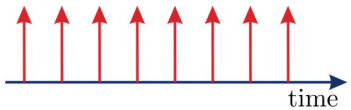
Introduction

Resource-aware control





- Resource-constrained control systems
  - Computation time on embedded systems
  - Actuator limitations (strain)
  - Network utilization in NCS
  - Battery power in WCS

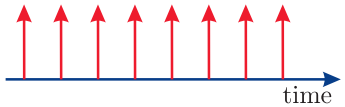


- Time-triggered periodic control: Inefficient usage of resources

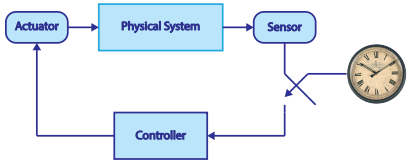
Introduction

Periodic or Aperiodic: That's the question!

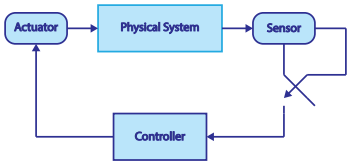
- Paradigm shift: Periodic control ⟶ Aperiodic control



- Only act when needed: bringing feedback in resource utilization



⟶



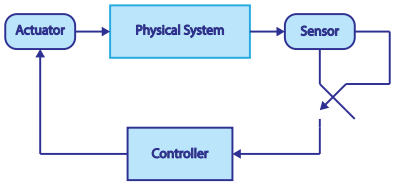
Introduction

Paradigm shift: Periodic control $\longrightarrow$ Aperiodic control

- Event-triggered control:

$$u(t) = \mathcal{K}(x(t_k)), \text{ when } t \in [t_k, t_{k+1})$$

$$t_{k+1} = \inf\{t > t_k \mid \mathcal{C}(x(t), x(t_k)) \geqslant 0\}$$



[1] Arzen, IFAC WC'99 [2] Astrom & Bernhardsson, IFAC WC'99 [3] Heemels *et al*, CEP'99

Introduction

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Paradigm shift: Periodic control $\longrightarrow$ Aperiodic control

- Event-triggered control:

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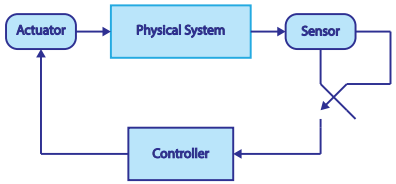


- Example event-triggering condition

$$\|x(t) - x(t_k)\| \geqslant \sigma \|x(t)\|$$

Introduction

Paradigm shift: Periodic control $\longrightarrow$ Aperiodic control

- Event-triggered control: reactive

$$u(t) = \mathcal{K}(x(t_k)), \text{ when } t \in [t_k, t_{k+1})$$

$$t_{k+1} = \inf\{t > t_k \mid \mathcal{C}(x(t), x(t_k)) \geqslant 0\}$$

- Self-triggered control: proactive

$$u(t) = \mathcal{K}(x(t_k)), \text{ when } t \in [t_k, t_{k+1})$$

$$t_{k+1} = t_k + \mathcal{M}(x(t_k))$$

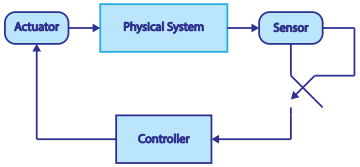
Outline

- Basic setup state-feedback ETC: $\|x(t) - x(t_k)\| \geqslant \sigma \|x(t)\|$
- Hybrid systems
- Challenges
  - Performance/Robustness w.r.t. disturbances & Zeno-freeness
  - Output-based (& Decentralized)
- Alternative event-triggered controllers
  - Relative, absolute and mixed event generators
  - Periodic event-triggered control
  - Time regularisation
  - Dynamic event generators
- Application to vehicle platooning
- Conclusions & What's next?

Basic ETC setup

- Linear system

$$\dot{x}(t) = Ax(t) + Bu(t)$$



- Linear state feedback

$$u(t) = Kx(t), \quad t \in \mathbb{R}_{\geqslant 0}$$

- Ideal loop: $\dot{x}(t) = (A + BK)x(t)$
- Sampled-data control with execution times t_k, $k \in \mathbb{N}$ (ZOH)

$$u(t) = K\hat{x}(t) = Kx(t_k), \quad t \in [t_k, t_{k+1})$$

- Perturbation perspective: implementation-induced error

$$e(t) = x(t_k) - x(t) \text{ for } t \in [t_k, t_{k+1})$$

$$\dot{x}(t) = Ax(t) + BKx(t_k) = (A + BK)x(t) + BKe(t)$$

[1] Tabuada, *Event-triggered real-time scheduling of stabilizing control tasks*, TAC 2007

Basic ETC setup

- Perturbation perspective:

$$\dot{x}(t) \;=\; Ax(t) + BKx(t_k) = (A + BK)x(t) + BKe(t)$$

- Since $A + BK$ Hurwitz, quadratic Lyapunov function $V(x) = x^\top Px$ s.t.

$$\frac{d}{dt}V \leqslant -a^2\|x(t)\|^2 + \|e(t)\|^2$$

- Crux: Guarantee $\|e(t)\| \leqslant \rho a \cdot \|x(t)\|$ with $0 < \rho < 1$ s.t.

$$\frac{d}{dt}V \leqslant -a^2\|x(t)\|^2 + \|e(t)\|^2 \leqslant -(1-\rho^2)a^2\|x(t)\|^2$$

- Guarantee for Global Exponential Stability

$$t_{k+1} = \inf\{t > t_k \mid \|\underbrace{x(t_k) - x(t)}_{=e(t)}\| \geqslant \rho a \cdot \|x(t)\|\}$$

Event-triggered control

- Summary of event-triggered setup:
  - Linear system

$$\dot{x}(t) = Ax(t) + Bu(t)$$

  - Execution times t_k, $k \in \mathbb{N}$

$$t_{k+1} = \inf\{t > t_k \mid \|x(t_k) - x(t)\| \geqslant \rho a \cdot \|x(t)\|\}$$

  - Control law:

$$u(t) = Kx(t_k), \quad t \in [t_k, t_{k+1})$$

- Global exponential stability (GES)

Event-triggered control

- Summary of event-triggered setup:
  - Linear system

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  - Control law:

$$u(t) = Kx(t_k), \quad t \in [t_k, t_{k+1})$$

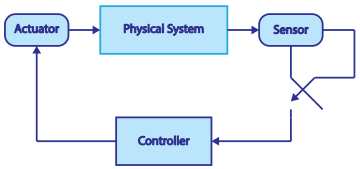
- Global exponential stability (GES)

- Question: Which important issue should we still verify?

Basic ETC setup

- Linear system

$$\dot{x}(t) = Ax(t) + Bu(t)$$



- Linear state feedback (ZOH)

$$u(t) = Kx(t_k), \quad t \in [t_k, t_{k+1})$$

- Execution times: $t_{k+1} = \inf\{t > t_k \mid \| \underbrace{x(t_k) - x(t)}_{=e(t)} \| \geqslant \sigma \|x(t)\|\}$
- Properties established in [1]:
  - Global exponential stability (GES) when σ suff. small
  - Global positive lower bound on minimal inter-event time (MIET)

$$\inf\{t_{k+1} - t_k \mid k \in \mathbb{N}\} > \tau_{\min} > 0$$

- Improved designs for GES/$\mathcal{L}_\infty$-gain via hybrid system analysis [2]

[1] Tabuada, *Event-triggered real-time scheduling of stabilizing control tasks*, TAC 2007
[2] Donkers, Heemels, *Output-Based Event-Triggered Control with Guaranteed L_∞-gain ...*, TAC 2012

Hybrid systems (side trip)

- Perturbation perspective:

$$\dot{x}(t) \;=\; Ax(t) + BKx(t_k) = (A + BK)x(t) + BKe(t)$$

- Execution times t_k, $k \in \mathbb{N}$

$$t_{k+1} = \inf\{t > t_k \mid \| \underbrace{x(t_k) - x(t)}_{=e(t)} \| \geqslant \sigma \|x(t)\|\}$$

- Hybrid system perspective [1,2] based on jump-flow models [3]:

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} x \\ e \end{bmatrix} = \begin{bmatrix} (A+BK)x + BKe \\ -(A+BK)x - BKe \end{bmatrix}, \qquad \text{when } \|e\|^2 \leqslant \sigma^2 \|x\|^2$$

$$\begin{bmatrix} x^+ \\ e^+ \end{bmatrix} = \begin{bmatrix} x \\ 0 \end{bmatrix}, \qquad \text{when } \|e\|^2 \geqslant \sigma^2 \|x\|^2$$

[1] Donkers, Heemels, *Output-Based Event-Triggered Control ...*, TAC 2012 & CDC 2010
[2] Postoyan, Anta, Nesic, Tabuada, *A unifying Lyapunov-based framework ...*, CDC-ECC 2011
[3] Goebel, Sanfelice, Teel, *Hybrid Dynamical Systems*, Princeton, 2012.

ETC based on feedback

Hybrid system perspective (side trip)

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} x \\ e \end{bmatrix} = \begin{bmatrix} (A+BK)x + BKe \\ -(A+BK)x - BKe \end{bmatrix} \qquad \text{when } \|e\|^2 \leqslant \sigma^2 \|x\|^2$$

$$\begin{bmatrix} x^+ \\ e^+ \end{bmatrix} = \begin{bmatrix} x \\ 0 \end{bmatrix} \qquad \text{when } \|e\|^2 \geqslant \sigma^2 \|x\|^2$$

or compactly with $\xi = \begin{bmatrix} x \\ e \end{bmatrix}$ $\quad \begin{cases} \dot{\xi} = \Phi\xi, \text{ when } \xi^\top Q \xi \leqslant 0 \\ \xi^+ = J\xi, \text{ when } \xi^\top Q \xi \geqslant 0 \end{cases}$

ETC based on feedback

Hybrid system perspective (side trip)

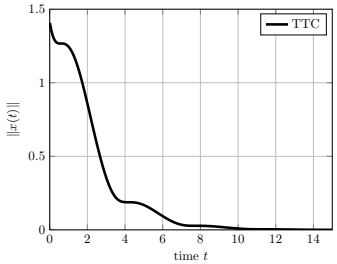
$$\begin{aligned} \dot{\xi} &= \Phi\xi \quad \text{when } \xi^\top Q \xi \leqslant 0 \\ \xi^+ &= J\xi \quad \text{when } \xi^\top Q \xi \geqslant 0 \end{aligned}$$

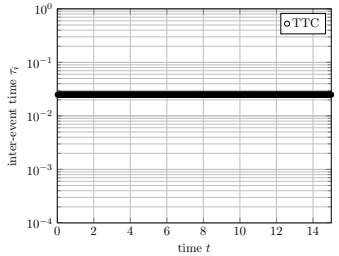
- Stability analysis using hybrid tools [1,2]: $V(\xi) = \xi^\top P \xi$
  - $\frac{d}{dt}V(\xi) < 0$ when $\xi^\top Q \xi \leqslant 0$
  - $V(J\xi) \leqslant V(\xi)$ when $\xi^\top Q \xi \geqslant 0$
- Linear matrix inequalities: if there are $\alpha, \beta \geqslant 0$ s.t.
  - $\Phi^\top P + P\Phi - \alpha Q \prec 0$
  - $J^\top P J - P + \beta Q \preceq 0$
- Guarantee for GES (extended ideas apply for $\mathcal{L}_\infty$-gains)
- Never more conservative than perturbation approach [1]

[1] Donkers, Heemels, *Output-based event-triggered control with guaranteed L_∞-gain*, TAC 2012 & CDC 2010
[2] Goebel, Sanfelice, Teel, *Hybrid Dynamical Systems: Modeling, Stability and Robustness*, Princeton, 2012.

Example 1: State feedback control

- Consider $\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$ and $u(t) = \begin{bmatrix} 1 & -4 \end{bmatrix} x(t_k)$
- TTC: $t_k = k \cdot 0.025$
- ETC: $t_k = t \iff \|e(t)\| \geqslant 0.05\|x(t)\|$ MIET = 0.025





Illustrative Example 15/52

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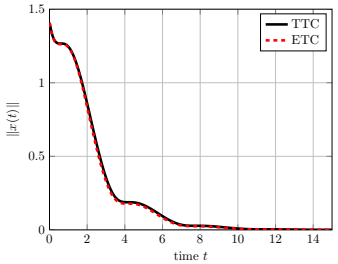

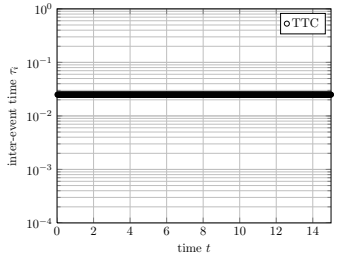

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Illustrative Example 15/52

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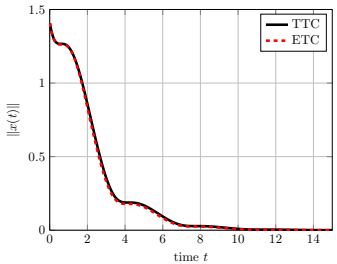

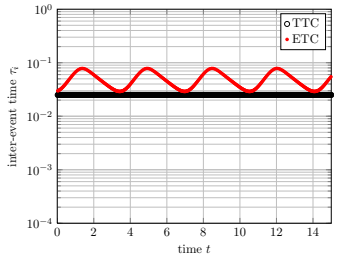

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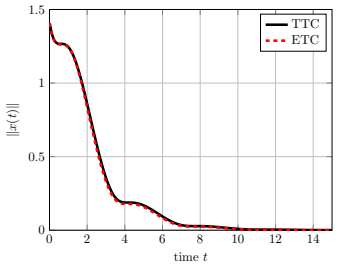

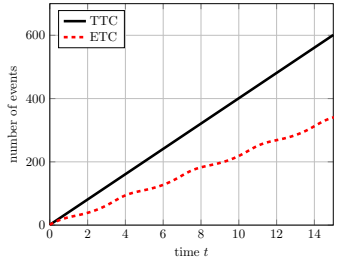

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Illustrative Examples

Example 1: Comparison P and HS approach

- Consider $\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$ and $u(t) = \begin{bmatrix} 1 & -4 \end{bmatrix} x(t_k)$
- Example taken from [1]
- We look for largest σ giving GES: $\|e\|^2 \leqslant \sigma^2 \|x\|^2$ [2]

	σ^2	MIET
P: Results from [1]	0.0030	0.0318
P: By minimising the $\mathcal{L}_2$-gain	0.0273	0.0840
Hybrid System	0.0588	0.1136

- PS: via minimising $\mathcal{L}_2$-gain: maximise a (note $\sigma = \rho a$)

$$\dot{V} \leqslant -a^2 \|x(t)\|^2 + \|e(t)\|^2 \quad \text{for } \dot{x} = (A + BK)x + BKe$$

- ETM:

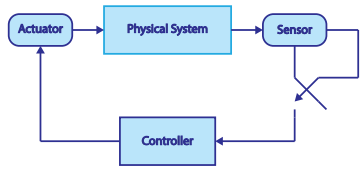
$$t_{k+1} = \inf\{t > t_k \mid \| \underbrace{x(t_k) - x(t)}_{=e(t)} \| \geqslant \rho a \cdot \|x(t)\|\}$$

[1] Tabuada, TAC '07 [2] Donkers, Heemels, CDC10 & TAC12

Summary

- Linear system

$$\dot{x}(t) = Ax(t) + Bu(t)$$



- Linear state feedback (ZOH)

$$u(t) = Kx(t_k), \quad t \in [t_k, t_{k+1})$$

- Execution times: $t_{k+1} = \inf\{t > t_k \mid \| \underbrace{x(t_k) - x(t)}_{=e(t)} \| \geqslant \sigma \|x(t)\|\}$
- Properties established in [1]:
  - Global exponential stability (GES) when σ suff. small
  - Global positive lower bound on minimal inter-event time (MIET)

$$\inf\{t_{k+1} - t_k \mid k \in \mathbb{N}\} > \tau_{\min} > 0$$

- Improved designs for GES/$\mathcal{L}_\infty$-gain via hybrid system analysis [2]

[1] Tabuada, *Event-triggered real-time scheduling of stabilizing control tasks*, TAC 2007
[2] Donkers, Heemels, *Output-Based Event-Triggered Control with Guaranteed L_∞-gain ...*, TAC 2012

Challenges

- Performance/Robustness w.r.t. disturbances
- Output-based (& Decentralized)

Disturbances in ETC

Illustrative example

- Consider $\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$ and $u(t) = \begin{bmatrix} 1 & -4 \end{bmatrix} x(t_k)$
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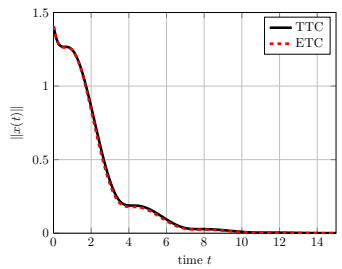

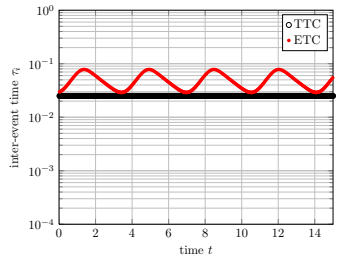

Borgers, Heemels, *Event-Separation Properties of Event-Triggered Control Systems*, TAC 2014

Illustrative example

- Consider $\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + w$ and $u(t) = \begin{bmatrix} 1 & -4 \end{bmatrix} x(t_k)$
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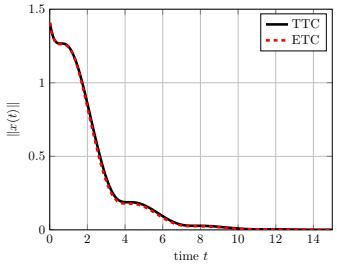

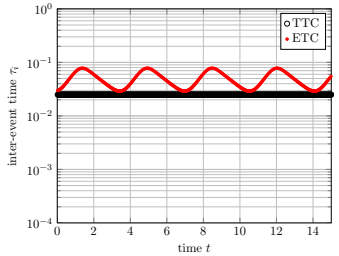

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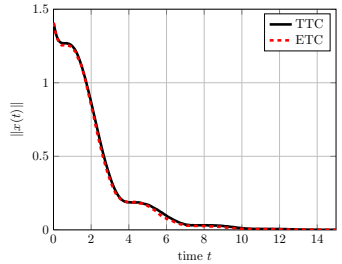

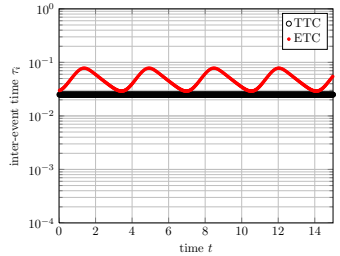

Borgers, Heemels, *Event-Separation Properties of Event-Triggered Control Systems*, TAC 2014

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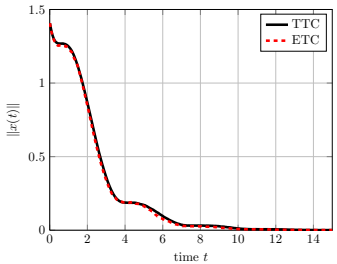
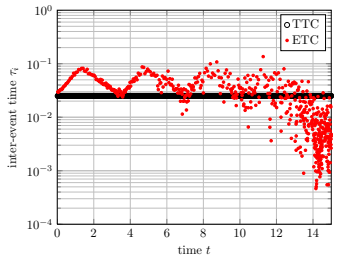
Illustrative example

- Consider $\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + w$ and $u(t) = \begin{bmatrix} 1 & -4 \end{bmatrix} x(t_k)$
- TTC: $t_k = k \cdot 0.025$
- ETC: $t_k = t \iff \|e(t)\| \geqslant 0.05 \|x(t)\|$





Borgers, Heemels, *Event-Separation Properties of Event-Triggered Control Systems*, TAC 2014

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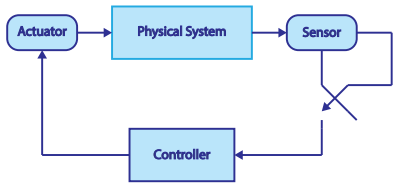
Illustrative example



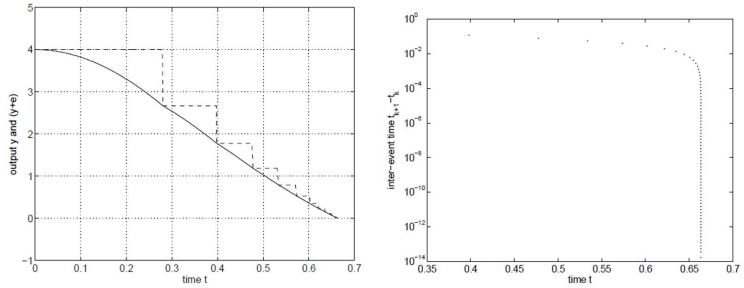
- Consider

$$\begin{cases} \dot{x}_p = \begin{bmatrix} 1 & -1 \\ 10 & -1 \end{bmatrix} x_p + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u \\ y = \begin{bmatrix} 1 & 0 \end{bmatrix} x_p \end{cases} \qquad u(t) = -2y(t_k)$$

- ETM: $\|y(t) - y(t_k)\|^2 > \sigma^2 \|y(t)\|^2$
- Parameter: $\sigma^2 = 0.5$

Output-based ETC

Illustrative example



- Minimal inter-event time (MIET) is zero! (Zeno behavior)

Donkers, Heemels, *Output-Based Event-Triggered Control with Guaranteed L_∞-gain and Improved and Decentralised Event-Triggering*, TAC 2012

Event-triggered control schemes

- Relative: $\|y-\hat{y}\| \geqslant \sigma\|y\|$ [1]
- Absolute: $\|y-\hat{y}\| \geqslant \delta$ [2-4]
- Mixed: $\|y-\hat{y}\| \geqslant \sigma\|y\|+\delta$ [5]

[1] Tabuada, *Event-triggered real-time scheduling of stabilizing control tasks*, TAC 2007
[2] Yook, Tilbury, Soparkar, *Trading computation for bandwidth: Reducing communication in distributed control systems using state estimators*, TCST 2002
[3] Miskowicz, *Send-on-delta concept: An event-based data-reporting strategy*, Sensors, 2006
[4] Lunze and Lehmann, *A state-feedback approach to event-based control*, Automatica, 2010
[5] Donkers, Heemels, *Output-Based Event-Triggered Control with Guaranteed L_∞-gain ...*, TAC 2012

Movie ETC in action

Inverted pendulum

Disturbances in ETC

Event-separation properties / Zeno-freeness

- Consider $\dot{x} = Ax + Bu + w$ and $u(t) = Kx(t_k) = K(x(t)+e(t))$
- Execution times:

$$t_{k+1} = \inf\{t > t_k \mid \|\underbrace{x(t_k)-x(t)}_{=e(t)}\| \geqslant \sigma\|x(t)\|+\delta\}$$

$\rightarrow$ MIET $\tau(x_0,w)$ dependent on x_0 and w: $\tau(x_0,w) = \inf_{k\in\mathbb{N}}(t_{k+1}-t_k)$

- Event-separation properties (nominal)
  - Global ESP: $\inf_{x_0\in\mathbb{R}^n}\tau(x_0,0) > 0$
  - Semi-global ESP: for compact $X_0 \subset \mathbb{R}^n$: $\inf_{x_0\in X_0}\tau(x_0,0) > 0$
  - Local ESP: for each $x_0 \in \mathbb{R}^n$: $\tau(x_0,0) > 0$

Borgers, Heemels, *Event-Separation Properties of Event-Triggered Control Systems*, TAC 2014

Disturbances in ETC

Event-separation properties / Zeno-freeness

- Consider $\dot{x} = Ax + Bu + w$ and $u(t) = Kx(t_k) = K(x(t) + e(t))$
- Execution times:

$$t_{k+1} = \inf\{t > t_k \mid \| \underbrace{x(t_k) - x(t)}_{=e(t)} \| \geqslant \sigma\|x(t)\| + \delta\}$$

→ MIET $\tau(x_0, w)$ dependent on x_0 and w: $\tau(x_0, w) = \inf_{k\in\mathbb{N}} (t_{k+1} - t_k)$

- Event-separation properties (robust): there is $\varepsilon > 0$
  - Robust global: $\inf_{x_0\in\mathbb{R}^n,\ \|w\|_\infty<\varepsilon} \tau(x_0, w) > 0$
  - Robust semi-global: compact X_0: $\inf_{x_0\in X_0,\ \|w\|_\infty<\varepsilon} \tau(x_0, w) > 0$
  - Robust local: for each $x_0 \in \mathbb{R}^n$ and $\|w\|_\infty < \varepsilon$: $\tau(x_0, w) > 0$

Borgers, Heemels, *Event-Separation Properties of Event-Triggered Control Systems*, TAC 2014

Overview

State-feedback case

ETM	robust global	global	robust semi-global	semi-global	robust local	local
relative	×	✓	×	✓	×	✓
absolute	×	×	✓	✓	✓	✓
mixed	✓	✓	✓	✓	✓	✓

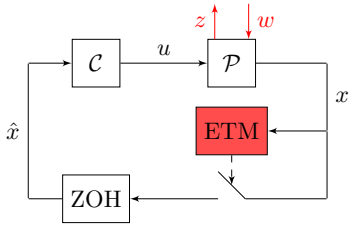
Output-feedback case

ETM	robust global	global	robust semi-global	semi-global	robust local	local
relative	×	×	×	×	×	×
absolute	×	×	✓	✓	✓	✓
mixed	×	×	✓	✓	✓	✓

- Relative triggering fragile. Zero robustness
- Mixed or absolute effective (semi-global)
- However, only practical stability / ultimate boundedness (no GAS)
- Challenge: What about robust global ESP and GAS/$\mathcal{L}_2$-gains?

Borgers, Heemels, *Event-Separation Properties of Event-Triggered Control Systems*, TAC 2014

Objectives



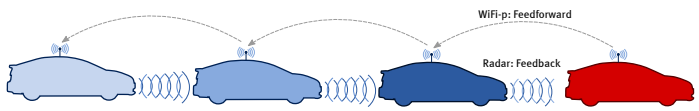
- Guaranteed control performance ($\mathcal{L}_2$−gain) from disturbance w to output $z = q(x, w)$:

$$\|z\|_{\mathcal{L}_2} \leqslant \beta(|\xi_0|) + \gamma\|w\|_{\mathcal{L}_2} \text{ with } \|z\|_{\mathcal{L}_2} = \sqrt{\int_0^\infty \|z(t)\|^2 dt}$$

- Global asymptotic stability (GAS) in absence of disturbances
- Robust positive "minimal inter-event time" (τ_{miet})
- Reduced communication w.r.t. time-triggered control

Motivation

Cooperative Adaptive Cruise Control



- String stability: disturbance attenuation along the vehicle string $\gamma \leqslant 1$

$$\|z\|_{\mathcal{L}_2} \leqslant \beta(|\xi_0|) + \gamma\|w\|_{\mathcal{L}_2} \text{ with } \|z\|_{\mathcal{L}_2} = \sqrt{\int_0^\infty \|z(t)\|^2 dt}$$

- Communication resources limited → event-triggered communication

Event-triggered control schemes

Time regularisation:

- Periodic Event-Triggered Control (PETC) [6-9]

$$t_{k+1} = \inf\{t > t_k \mid \|y(t) - \hat{y}(t)\| > \sigma\|y(t)\| \wedge t = kh,\ k \in \mathbb{N}\}$$

- Enforcing minimal inter-event time [7,9-11]

$$t_{k+1} = \inf\{t > t_k + T \mid \|y(t) - \hat{y}(t)\| > \sigma\|y(t)\|\}$$

[6] Arzen, *A simple event-based PID controller*, IFAC 1999
[7] Heemels, Sandee, van den Bosch, *Analysis of event-driven controllers for linear systems*, IJC 2008
[8] Heemels, Donkers, Teel, *Periodic Event-Triggered Control for Linear Systems*, TAC 2013
[9] Henningsson, Johannesson, Cervin, *Sporadic event-based control of first-order linear stochastic ..*, Aut. 2008
[10] Tallapragada, Chopra, *Event-triggered dynamic output feedback control for LTI systems*, CDC 2012
[11] Tallapragada, Chopra, *Event-triggered decentralized dynamic output .. LTI systems*, NECSYS 2012

Time regularized ETC

- Periodic Event-Triggered Control (PETC)

$$t_{k+1} = \inf\{t > t_k \mid \|y - \hat{y}\| > \sigma\|y\| \wedge t = kh,\ k \in \mathbb{N}\}$$

- Enforcing minimal inter-event time

$$t_{k+1} = \inf\{t > t_k + T \mid \|y - \hat{y}\| > \sigma\|y\|\}$$

Output-feedback case

ETM	robust global	global	robust semi-global	semi-global	robust local	local
relative	×	×	×	×	×	×
absolute	×	×	✓	✓	✓	✓
mixed	×	×	✓	✓	✓	✓
time-regu	✓	✓	✓	✓	✓	✓

PETC

- Periodic Event-Triggered Control (PETC)

$$t_{k+1} = \inf\{t > t_k \mid \|y - \hat{y}\| > \sigma\|y\| \wedge t = kh,\ k \in \mathbb{N}\}$$

- Hybrid system analysis: GAS & finite $\mathcal{L}_2$-gains [1,2]
- Implementation advantages:
  - Guaranteed (reasonable) minimal inter-event time
  - Only time-periodic verification of event-triggering conditions
  - More in line with time-sliced architectures

[1] Heemels, Donkers, Teel, *Periodic Event-Triggered Control for Linear Systems*, TAC 2013
[2] Heemels, Donkers, *Model-based Periodic Event-Triggered Control for Linear Systems*, Automatica 2013

PETC

Hybrid systems formulation

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix}\xi\\ \tau\end{bmatrix} = \begin{bmatrix}A\xi + Bw\\ 1\end{bmatrix}, \text{ when } \tau \in [0, h],$$

$$\begin{bmatrix}\xi^+\\ \tau^+\end{bmatrix} = \begin{cases}\begin{bmatrix}J_1\xi\\ 0\end{bmatrix}, & \text{when } \xi^\top Q\xi > 0,\ \tau = h\\ \begin{bmatrix}J_2\xi\\ 0\end{bmatrix}, & \text{when } \xi^\top Q\xi \leqslant 0,\ \tau = h\end{cases}$$

$$z = C\xi + Dw$$

Hybrid systems formulation

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix}\xi\\ \tau\end{bmatrix} = \begin{bmatrix}A\xi + Bw\\ 1\end{bmatrix}, \text{ when } \tau \in [0,h],$$

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$$z = C\xi + Dw$$

- In case $w = 0$ and interested in stability only
- Discretize at kh, $k \in \mathbb{N}$ (just before jump) leading to discrete-time PWL system [1,2,3]

$$\xi_{k+1} = \begin{cases}e^{Ah}J_1\xi_k, & \text{when } \xi_k^\top Q\xi_k > 0\\ e^{Ah}J_2\xi_k, & \text{when } \xi_k^\top Q\xi_k \leqslant 0\end{cases}$$

[1] Heemels, Donkers, Teel, *Periodic Event-Triggered Control for Linear Systems*, TAC 2013
[2] Heemels, Donkers, *Model-based Periodic Event-Triggered Control for Linear Systems*, Automatica 2013
[3] Heemels, Sandee, van den Bosch, *Analysis of event-driven controllers for linear systems*, IJC 2008

Hybrid systems formulation

- Including intersample-behavior, e.g., for $\mathcal{L}_2$-gain analysis

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix}\xi\\ \tau\end{bmatrix} = \begin{bmatrix}A\xi + Bw\\ 1\end{bmatrix}, \text{ when } \tau \in [0,h],$$

$$\begin{bmatrix}\xi^+\\ \tau^+\end{bmatrix} = \begin{cases}\begin{bmatrix}J_1\xi\\ 0\end{bmatrix}, & \text{when } \xi^\top Q\xi > 0,\ \tau = h\\ \begin{bmatrix}J_2\xi\\ 0\end{bmatrix}, & \text{when } \xi^\top Q\xi \leqslant 0,\ \tau = h\end{cases}$$

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Hybrid systems formulation

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix}\xi\\ \tau\end{bmatrix} = \begin{bmatrix}A\xi + Bw\\ 1\end{bmatrix}, \text{ when } \tau \in [0,h]$$

$$\begin{bmatrix}\xi^+\\ \tau^+\end{bmatrix} = \begin{bmatrix}\phi(\xi)\\ 0\end{bmatrix}, \qquad \text{when } \tau = h$$

$$z = C\xi + Dw$$

Hybrid systems formulation

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix}\xi\\ \tau\end{bmatrix} = \begin{bmatrix}A\xi + Bw\\ 1\end{bmatrix}, \text{ when } \tau \in [0,h]$$

$$\begin{bmatrix}\xi^+\\ \tau^+\end{bmatrix} = \begin{bmatrix}\phi(\xi)\\ 0\end{bmatrix}, \qquad \text{when } \tau = h$$

$$z = C\xi + Dw$$

- $\mathcal{L}_2$-contractive: There are $\gamma_0 \in [0,1)$ and a $\mathcal{K}$-function β s.t.

$$\|z\|_{\mathcal{L}_2} \leqslant \beta(|\xi_0|) + \gamma_0\|w\|_{\mathcal{L}_2} \text{ with } \|z\|_{\mathcal{L}_2} = \sqrt{\int_0^\infty \|z(t)\|^2 dt}$$

Lifting-based approach

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix}\xi\\ \tau\end{bmatrix} = \begin{bmatrix}A\xi + Bw\\ 1\end{bmatrix}, \quad \text{when } \tau \in [0,h]$$

$$\begin{bmatrix}\xi^+\\ \tau^+\end{bmatrix} = \begin{bmatrix}\phi(\xi)\\ 0\end{bmatrix}, \quad \text{when } \tau = h$$

$$z = C\xi + Dw$$

$$\begin{aligned}\bar{\xi}_{k+1} &= A_d\phi(\bar{\xi}_k) + B_d v_k\\ r_k &= C_d\phi(\bar{\xi}_k)\end{aligned}$$

Main result: The hybrid system is internally stable and $\mathcal{L}_2$-contractive iff the discrete-time nonlinear system is internally stable and ℓ_2-contractive.

- ℓ_2-contractive: there is $\gamma_0 \in [0,1)$ s.t.

$$\|r\|_{\ell_2} \leqslant \beta(|\bar{\xi}_0|) + \gamma_0\|v\|_{\ell_2} \text{ with } \|r\|_{\ell_2}^2 = \sum_{k=0}^{\infty}|r_k|^2$$

[1] Heemels, Dullerud, Teel, *$\mathcal{L}_2$-gain Analysis for a Class of Hybrid Systems with Applications to Reset and Event-triggered Control: A Lifting Approach*

Lifting-based approach

34/52

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix}\xi\\ \tau\end{bmatrix} = \begin{bmatrix}A\xi + Bw\\ 1\end{bmatrix}, \quad \text{when } \tau \in [0,h]$$

$$\begin{bmatrix}\xi^+\\ \tau^+\end{bmatrix} = \begin{bmatrix}\phi(\xi)\\ 0\end{bmatrix}, \quad \text{when } \tau = h$$

$$z = C\xi + Dw$$

$$\begin{aligned}\bar{\xi}_{k+1} &= A_d\phi(\bar{\xi}_k) + B_d v_k\\ r_k &= C_d\phi(\bar{\xi}_k)\end{aligned}$$

Main result: The hybrid system is internally stable and $\mathcal{L}_2$-contractive iff the discrete-time nonlinear system is internally stable and ℓ_2-contractive.

- Lifting with verifiable conditions without linearity
- For PETC *piecewise linear* system $\longrightarrow$ contractivity/stability analysis via LMIs using piecewise quadratic Lyapunov functions

[1] Heemels, Dullerud, Teel, *$\mathcal{L}_2$-gain Analysis for a Class of Hybrid Systems with Applications to Reset and Event-triggered Control: A Lifting Approach*

/department of mechanical engineering

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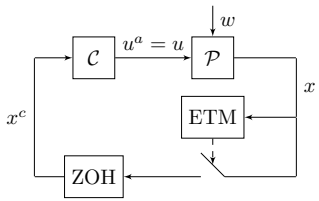
Time regularisation: Example



$$\mathcal{P}: \dot{x} = \begin{bmatrix}0 & 1\\ -2 & 3\end{bmatrix}x + \begin{bmatrix}0\\ 1\end{bmatrix}u + w$$

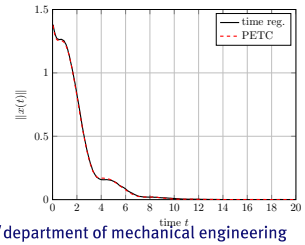
$$\mathcal{C}: u = \begin{bmatrix}1 & -4\end{bmatrix}x^c$$

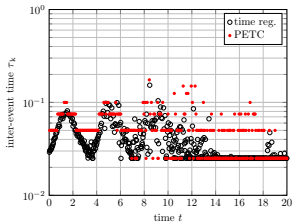
- Periodic Event-Triggered Control (PETC)

$$t_{k+1} = \inf\{t > t_k \mid \|x^c(t) - x(t)\| > \sigma\|x(t)\| \wedge t = kh,\ k \in \mathbb{N}\}$$

- Enforcing minimal inter-event time

$$t_{k+1} = \inf\{t > t_k + T \mid \|x^c(t) - x(t)\| > \sigma\|x(t)\|\}$$





Dynamic event-triggered control

- Static event generator: $t_{k+1} := \inf\{t > t_k + T | C(x(t), e(t)) > 0\}$

Dynamic event generator [1,2,3]

$$\dot{\eta} = \Psi(x, e, \eta)$$
$$t_{k+1} := \inf\{t > t_k + T \mid \eta(t) < 0\}$$

- How to find Ψ and T?

[1] Postoyan et al., "Event-triggered and self-triggered stabilization ...," CDC 2011
[2] Girard, "Dynamic triggering mechanisms for event-triggered control," TAC 2015
[3] Dolk, Borgers, Heemels, "Dynamic Event-triggered Control...," CDC 2014

Recap: Design relative triggering

- Perturbation perspective:

$$\dot{x}(t) \;=\; Ax(t) + BKx(t_k) = (A+BK)x(t) + BKe(t)$$

- Since $A+BK$ Hurwitz, quadratic Lyapunov function $V(x) = x^\top Px$

$$\frac{d}{dt}V \leqslant -a^2\|x(t)\|^2 + \|e(t)\|^2$$

- Crux: Guarantee $\|e(t)\| \leqslant \rho a \cdot \|x(t)\|$ with $0 < \rho < 1$ s.t.

$$\frac{d}{dt}V \leqslant -a^2\|x(t)\|^2 + \|e(t)\|^2 \leqslant -(1-\rho^2)a^2\|x(t)\|^2$$

- Guarantee for Global Exponential Stability

$$t_{k+1} = \inf\{t > t_k \mid \|\underbrace{x(t_k) - x(t)}_{=e(t)}\| \geqslant \rho a \cdot \|x(t)\|\}$$

- Zeno-free: There is $T > 0$ such that $t_{k+1} - t_k \geqslant T$ for all $k \in \mathbb{N}$.

Basic design dETM

- Since $A+BK$ Hurwitz, quadratic Lyapunov function $V(x) = x^\top Px$

$$\frac{d}{dt}V \leqslant -a^2\|x(t)\|^2 + \|e(t)\|^2$$

- Now consider $\dot{\eta} = \Psi(x,e,\eta)$ and LF $U(x,\eta) = V(x) + \eta$ [2] :

$$\frac{d}{dt}U \leqslant -a^2\|x\|^2 + \|e\|^2 + \Psi$$

- To get $\frac{d}{dt}U \leqslant -(1-\rho^2)a^2\|x\|^2 - \varepsilon\eta$ for some $\varepsilon > 0$ we require

$$-a^2\|x\|^2 + \|e\|^2 + \Psi = -(1-\rho^2)a^2\|x\|^2 - \varepsilon\eta$$

  and thus $\dot{\eta} = \Psi(x,e,\eta) = \rho^2a^2\|x\|^2 - \varepsilon\eta - \|e\|^2$
- Now $t_{k+1} := \inf\{t > t_k + T \mid \eta(t) < 0\}$, $\eta(0) = 0$ and $t_0 = 0$:
  - $\eta(t) \geqslant 0$ for $t \in \mathbb{R}_{\geqslant 0}$ and thus U positive definite
  - $\frac{d}{dt}U \leqslant -(1-\rho^2)a^2\|x\|^2 - \varepsilon\eta$ and thus GES
- Never triggers before the static version!!

Dynamic event-triggered control

- Static event generator: $t_{k+1} := \inf\{t > t_k + T | C(x(t), e(t)) > 0\}$

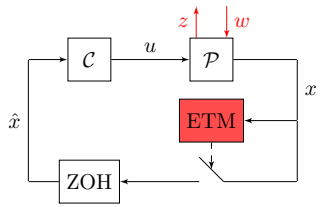
Dynamic event generator [1,2,3]

$$\dot{\eta} = \Psi(x,e,\eta)$$
$$t_{k+1} := \inf\{t > t_k + T \mid \eta(t) < 0\}$$

- [1,2] design for $w = 0$ (no disturbances)
- Recently, [3] new design methodology for output-based decentralized triggering under disturbances ($\mathcal{L}_p$-gain)

[1] Postoyan et al., "Event-triggered and self-triggered stabilization ...," CDC 2011
[2] Girard, "Dynamic triggering mechanisms for event-triggered control," TAC 2015
[3] Dolk, Borgers, Heemels, "Dynamic Event-triggered Control...," CDC 2014 and TAC?

Dynamic ETC: Example



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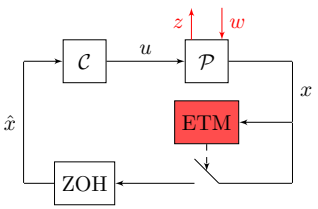
$$\mathcal{C} : u = \begin{bmatrix} 1 & -4 \end{bmatrix} x^c$$

Case study: $\mathcal{L}_2$-gain $\theta = 4$ from input w to state x: $\tau_{miet} = 9.1 \cdot 10^{-3}$

- *Dynamic* event generator $t_{k+1} := \inf\{t > t_k + \tau_{miet} \mid \eta(t) < 0\}$
- *Static* event generator: $t_{k+1} := \inf\{t > t_k + \tau_{miet} \mid \Psi(x,e,\tau,\eta) < 0\}$

Dynamic ETC: Example

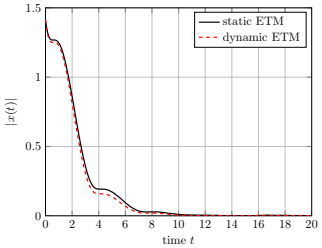


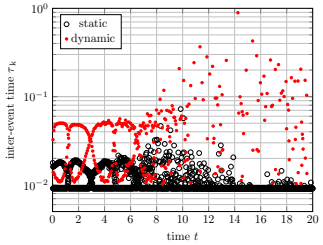
$$\mathcal{P}: \dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + w$$

$$\mathcal{C}: u = \begin{bmatrix} 1 & -4 \end{bmatrix} x^c$$

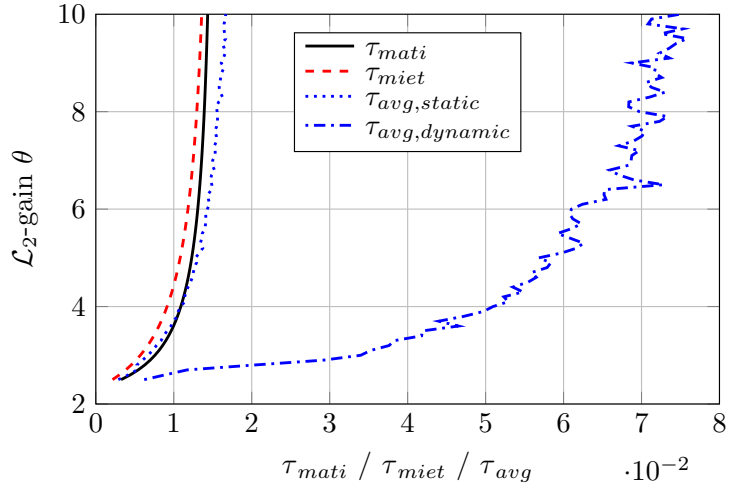
Case study: $\mathcal{L}_2$-gain $\theta = 4$ from input w to state x: $\tau_{miet} = 9.1 \cdot 10^{-3}$

- *Dynamic* event generator $t_{k+1} := \inf\{t > t_k + \tau_{miet} \mid \eta(t) < 0\}$
- *Static* event generator: $t_{k+1} := \inf\{t > t_k + \tau_{miet} \mid \Psi(x, e, \tau, \eta) < 0\}$



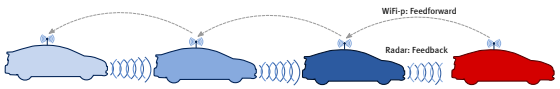


Dynamic ETC: Example

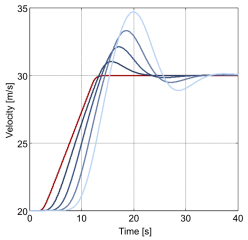


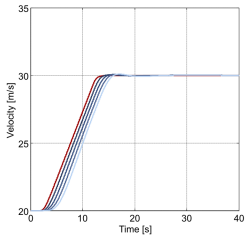
Motivation

Cooperative Adaptive Cruise Control

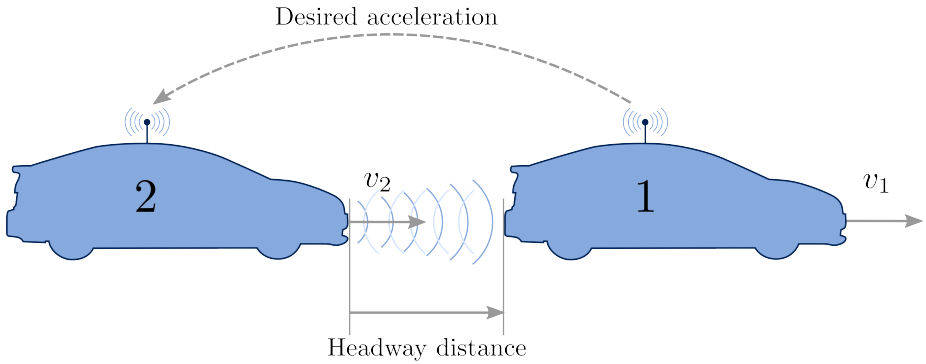


- String stability: disturbance attenuation along the vehicle string
  - $\mathcal{L}_p$-gain$\leqslant 1$
- Communication resources limited $\rightarrow$ event-triggered communication

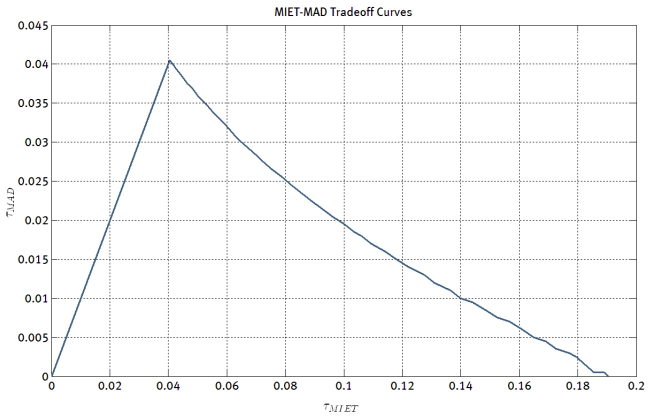




Cooperative Adaptive Cruise Control



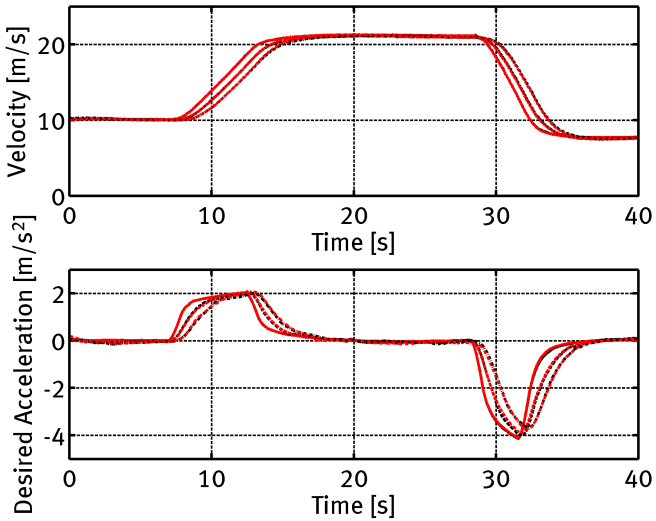
CACC



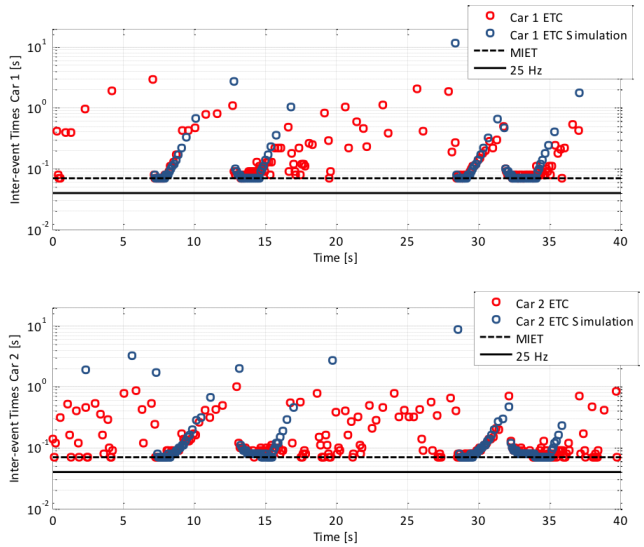
- Headway time: 0.6 seconds
- MIET: 0.07 seconds

$\longrightarrow$ MOVIE

CACC



CACC



Conclusions

- Event-triggered control: A new resource-aware control paradigm
- Several ETC algorithms discussed with their own tools (hybrid)
- Challenges
  - Performance / Robustness w.r.t. disturbances
  - Output-based & decentralized event generators
  - Constrained systems (MPC)
  - Implementation and Applications
  - Better than periodic time-triggered control
  - Improved analysis and design tools: MIET, average inter-execution times, $\mathcal{L}_p$-gain, etc.
- Many interesting practical and theoretical issues open in this appealing research field

- More info: http://www.heemels.tue.nl

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NWO Netherlands Organisation for Scientific Research

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- More info: http://www.heemels.tue.nl

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