Resource-aware control

Maurice Heemels

Introduction

Standard digital control loop

--- All control tasks executed periodically and triggered by time

Resource-aware control

- Resource-constrained control systems
  - Computation time on embedded systems
  - Actuator limitations (strain)
  - Network utilization in NCS
  - Battery power in WCS

- Time-triggered periodic control: Inefficient usage of resources

Periodic or Aperiodic: That’s the question!

- Paradigm shift: Periodic control $\rightarrow$ Aperiodic control
- Only act when needed: bringing feedback in resource utilization
### Introduction

**Paradigm shift:** Periodic control $\rightarrow$ Aperiodic control

- **Event-triggered control:**
  \[
  u(t) = \mathcal{K}(x(t_k)), \text{ when } t \in [t_k, t_{k+1})
  \]
  \[
  t_{k+1} = \inf \{ t > t_k | C(x(t), x(t_k)) \geq 0 \}
  \]

- **Self-triggered control:** proactive
  \[
  u(t) = \mathcal{K}(x(t_k)), \text{ when } t \in [t_k, t_{k+1})
  \]
  \[
  t_{k+1} = t_k + M(x(t_k))
  \]

---

**Outline**

- Basic setup state-feedback ETC: $\|x(t) - x(t_k)\| \geq \sigma\|x(t)\|$
- Hybrid systems
- Challenges
  - Performance/Robustness w.r.t. disturbances & Zeno-freeness
  - Output-based (& Decentralized)
- Alternative event-triggered controllers
  - Relative, absolute and mixed event generators
  - Periodic event-triggered control
  - Time regularisation
  - Dynamic event generators
- Application to vehicle platooning
- Conclusions & What’s next?
Basic ETC setup

- Linear system
  \[ \dot{x}(t) = Ax(t) + Bu(t) \]
- Linear state feedback
  \[ u(t) = K x(t), \quad t \in \mathbb{R}_{>0} \]
- Ideal loop: \( \dot{x}(t) = (A + BK)x(t) \)
- Sampled-data control with execution times \( t_k, k \in \mathbb{N} \) (ZOH)
  \[ u(t) = K \dot{x}(t) = K x(t_k), \quad t \in [t_k, t_{k+1}) \]
- Perturbation perspective: implementation-induced error
  \[ e(t) = x(t_k) - x(t) \text{ for } t \in [t_k, t_{k+1}) \]

\[ \dot{x}(t) = Ax(t) + BK x(t_k) = (A + BK)x(t) + BK e(t) \]


Event-triggered control

- Linear system
  \[ \dot{x}(t) = Ax(t) + Bu(t) \]
- Execution times \( t_k, k \in \mathbb{N} \)
  \[ t_{k+1} = \inf\{t > t_k \mid \|x(t_k) - x(t)\| \geq \rho a \cdot \|x(t)\|\} \]
- Control law:
  \[ u(t) = K x(t_k), \quad t \in [t_k, t_{k+1}) \]
- Global exponential stability (GES)

Basic ETC setup

- Perturbation perspective:
  \[ \dot{x}(t) = Ax(t) + BK x(t_k) = (A + BK)x(t) + BK e(t) \]
- Since \( A + BK \) Hurwitz, quadratic Lyapunov function \( V(x) = x^\top P x \) s.t.
  \[ \frac{d}{dt} V \leq -a^2 \|x(t)\|^2 + \|e(t)\|^2 \]
- Crux: Guarantee \( \|e(t)\| \leq \rho a \cdot \|x(t)\| \) with \( 0 < \rho < 1 \) s.t.
  \[ \frac{d}{dt} V \leq -a^2 \|x(t)\|^2 + \|e(t)\|^2 \leq -(1 - \rho^2)a^2 \|x(t)\|^2 \]
- Guarantee for Global Exponential Stability
  \[ t_{k+1} = \inf\{t > t_k \mid \|x(t_k) - x(t)\| \geq \rho a \cdot \|x(t)\|\} \]

Event-triggered control

- Linear system
  \[ \dot{x}(t) = Ax(t) + Bu(t) \]
- Execution times \( t_k, k \in \mathbb{N} \)
  \[ t_{k+1} = \inf\{t > t_k \mid \|x(t_k) - x(t)\| \geq \rho a \cdot \|x(t)\|\} \]
- Control law:
  \[ u(t) = K x(t_k), \quad t \in [t_k, t_{k+1}) \]
- Global exponential stability (GES)

- Question: Which important issue should we still verify?
Hybrid system perspective (side trip)

\[ \dot{x}(t) = Ax(t) + BKx(t_k) = (A + BK)x(t) + BKe(t) \]

- Perturbation perspective:

- Execution times: \( t_{k+1} = \inf\{ t > t_k \mid \| x(t_k) - x(t) \| \geq \sigma \| x(t) \| \} \)

Hybrid systems (side trip)

- Hybrid system perspective [1,2] based on jump-flow models [3]:

\[ \frac{d}{dt} \begin{bmatrix} x^+ \\ e^+ \end{bmatrix} = \begin{bmatrix} (A + BK)x + BKe \\ (A + BK)x - BKe \end{bmatrix}, \quad \text{when } \| e \|^2 \leq \sigma^2 \| x \|^2 \]

\[ \begin{bmatrix} x^+ \\ e^+ \end{bmatrix} = \begin{bmatrix} x \\ 0 \end{bmatrix}, \quad \text{when } \| e \|^2 \geq \sigma^2 \| x \|^2 \]

\[ \xi = \Phi \xi \quad \text{when } \xi^T Q \xi \leq 0 \]

\[ \xi^+ = J \xi \quad \text{when } \xi^T Q \xi \geq 0 \]

- Stability analysis using hybrid tools [1,2]: \( V(\xi) = \xi^T P \xi \)

- Linear matrix inequalities: if there are \( \alpha, \beta \geq 0 \) s.t.

- Guarantee for GES (extended ideas apply for \( L_\infty \)-gains)

- Never more conservative than perturbation approach [1]

References:
[1] Donkers, Heemels, Output-Based Event-Triggered Control with Guaranteed \( L_\infty \)-Gain, TAC 2012 & CDC 2010
**Example 1: State feedback control**

- Consider $\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$ and $u(t) = [1 -4]x(t_k)$

- TTC: $t_k = k \cdot 0.025$

- ETC: $t_k = t \iff \|e(t)\| \geq 0.05\|x(t)\|$ MIET = 0.025

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**Example 1: State feedback control**

- Consider $\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$ and $u(t) = [1 -4]x(t_k)$

- TTC: $t_k = k \cdot 0.025$

- ETC: $t_k = t \iff \|e(t)\| \geq 0.05\|x(t)\|$ MIET = 0.025
Illustrative Examples

Example 1: Comparison P and HS approach

- Consider $\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$ and $u(t) = [1 \ -4] x(t_k)$
- Example taken from [1]
- We look for largest $\sigma$ giving GES: $\|e\|^2 \leq \sigma^2 \|x\|^2$ [2]

<table>
<thead>
<tr>
<th>P: Results from [1]</th>
<th>P: By minimising the $L_2$-gain</th>
<th>Hybrid System</th>
</tr>
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<tr>
<td>$\sigma^2$</td>
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</table>

- PS: via minimising $L_2$-gain: maximise $a$ (note $\sigma = \rho a$)
  $$\dot{V} \leq -a^2 \|x(t)\|^2 + \|e(t)\|^2$$ for $\dot{x} = (A + BK)x + BK e$
- ETM:
  $$t_{k+1} = \inf \{t > t_k \mid \|x(t) - x(t_k)\| \geq \rho a \cdot \|x(t_k)\| \}$$

Challenges

- Performance/Robustness w.r.t. disturbances
- Output-based (& Decentralized)

Summary

- Linear system
  $$\dot{x}(t) = Ax(t) + Bu(t)$$
- Linear state feedback (ZOH)
  $$u(t) = Kx(t_k), \quad t \in [t_k, t_{k+1})$$
- Execution times:
  $$t_{k+1} = \inf \{t > t_k \mid \|x(t) - x(t_k)\| \geq \sigma \|x(t_k)\| \}$$
- Properties established in [1]:
  - Global exponential stability (GES) when $\sigma$ suff. small
  - Global positive lower bound on minimal inter-event time (MIET)
  $$\inf \{t_{k+1} - t_k \mid k \in \mathbb{N} \} > \tau_{\text{min}} > 0$$
- Improved designs for GES/$L_\infty$-gain via hybrid system analysis [2]

Illustrative example

- Consider $\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$ and $u(t) = [1 \ -4] x(t_k)$
- TTC: $t_k = k \cdot 0.025$
- ETC: $t_k = t \iff \|e(t)\| \geq 0.05 \|x(t)\|$
Illustrative example

- Consider $\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + w$ and $u(t) = \begin{bmatrix} 1 & -4 \end{bmatrix} x(t_k)$
- TTC: $t_k = k \cdot 0.025$
- ETC: $t_k = t \iff \|e(t)\| \geq 0.05\|x(t)\|$

Disturbances in ETC


Output-based ETC

- Consider $\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + w$ and $u(t) = \begin{bmatrix} 1 & -4 \end{bmatrix} x(t_k)$
- TTC: $t_k = k \cdot 0.025$
- ETC: $t_k = t \iff \|e(t)\| \geq 0.05\|x(t)\|$

Illustrative example

- Consider $\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + w$ and $u(t) = \begin{bmatrix} 1 & -4 \end{bmatrix} x(t_k)$
- TTC: $t_k = k \cdot 0.025$
- ETC: $t_k = t \iff \|e(t)\| \geq 0.05\|x(t)\|$

Actuator $\Rightarrow$ Physical System $\Rightarrow$ Sensor $\Rightarrow$ Controller

- Consider $\begin{cases} x_p = \begin{bmatrix} 1 & -1 \\ 10 & -1 \end{bmatrix} x_p + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u \\ y = \begin{bmatrix} 1 & 0 \end{bmatrix} x_p \end{cases}$ and $u(t) = -2y(t_k)$
- ETM: $\|y(t) - y(t_k)\|^2 > \sigma^2\|y(t_k)\|^2$
- Parameter: $\sigma^2 = 0.5$

Illustrative example

• Minimal inter-event time (MIET) is zero! (Zeno behavior)

Donkers, Heemels, Output-Based Event-Triggered Control with Guaranteed $L_{\infty}$-gain and Improved and Decentralised Event-Triggering, TAC 2012

Event-triggered control schemes

• Relative: $\|y - \hat{y}\| \geq \sigma \|y\|$ [1]
• Absolute: $\|y - \hat{y}\| \geq \delta$ [2-4]
• Mixed: $\|y - \hat{y}\| \geq \sigma \|y\| + \delta$ [5]


Event-separation properties / Zeno-freeness

• Consider $\dot{x} = Ax + Bu + w$ and $u(t) = K(x(t) + e(t))$

• Execution times:

$$t_{k+1} = \inf \{t > t_k \mid \| x(t_k) - x(t) \| \geq \sigma \|x(t)\| + \delta \}$$

→ MIET $\tau(x_0, w)$ dependent on $x_0$ and $w$:

$$\tau(x_0, w) = \inf_{k \in \mathbb{N}} (t_{k+1} - t_k)$$

• Event-separation properties (nominal)

− Global ESP: $\inf_{x_0 \in \mathbb{R}^n} \tau(x_0, 0) > 0$
− Semi-global ESP: for compact $X_0 \subset \mathbb{R}^n$: $\inf_{x_0 \in X_0} \tau(x_0, 0) > 0$
− Local ESP: for each $x_0 \in \mathbb{R}^n$: $\tau(x_0, 0) > 0$


Disturbances in ETC

Movie ETC in action

Inverted pendulum
Disturbances in ETC

Event-separation properties / Zeno-freeness

- Consider $\dot{x} = Ax + Bu + w$ and $u(t) = Kx(t) = K(x(t) + e(t))$
- Execution times:
  \[ t_{k+1} = \inf\{t > t_k \mid \|x(t_k) - x(t)\| \geq \sigma \|x(t)\| + \delta \} \]

\[ \rightarrow \text{MIET } \tau(x_0, w) \text{ dependent on } x_0 \text{ and } w: \quad \tau(x_0, w) = \inf_{k\in\mathbb{N}} (t_{k+1} - t_k) \]

- Event-separation properties (robust): there is $\varepsilon > 0$
  - Robust global: $\inf_{x_0 \in \mathbb{R}^n, \|w\|_\infty < \varepsilon} \tau(x_0, w) > 0$
  - Robust semi-global: compact $X_0$: $\inf_{x_0 \in X_0, \|w\|_\infty < \varepsilon} \tau(x_0, w) > 0$
  - Robust local: for each $x_0 \in \mathbb{R}^n$ and $\|w\|_\infty < \varepsilon$: $\tau(x_0, w) > 0$

Objectives

- Guaranteed control performance ($L_2$-gain) from disturbance $w$ to output $z = q(x, w)$:
  \[ \|z\|_{L_2} \leq \beta(\|x_0\|) + \gamma \|w\|_{L_2} \quad \text{with} \quad \|z\|_{L_2} = \sqrt{\int_0^\infty \|z(t)\|^2 dt} \]

- Global asymptotic stability (GAS) in absence of disturbances
- Robust positive “minimal inter-event time” ($\tau_{\text{mixed}}$)
- Reduced communication w.r.t. time-triggered control

Overview

State-feedback case

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<th>ETM</th>
<th>robust global</th>
<th>global</th>
<th>robust semi-global</th>
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Output-feedback case

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Motivation

Cooperative Adaptive Cruise Control

- String stability: disturbance attenuation along the vehicle string $\gamma \leq 1$
  \[ \|z\|_{L_2} \leq \beta(\|x_0\|) + \gamma \|w\|_{L_2} \quad \text{with} \quad \|z\|_{L_2} = \sqrt{\int_0^\infty \|z(t)\|^2 dt} \]

- Communication resources limited $\rightarrow$ event-triggered communication
Event-triggered control schemes

Time regularisation:

- Periodic Event-Triggered Control (PETC) [6-9]
  \[ t_{k+1} = \inf \{ t > t_k \mid \| y(t) - \hat{y}(t) \| > \sigma \| y(t) \| \land t = kh, \ k \in \mathbb{N} \} \]

- Enforcing minimal inter-event time [7,9-11]
  \[ t_{k+1} = \inf \{ t > t_k + T \mid \| y(t) - \hat{y}(t) \| > \sigma \| y(t) \| \} \]

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Time regularized ETC

- Periodic Event-Triggered Control (PETC)
  \[ t_{k+1} = \inf \{ t > t_k \mid \| y - \hat{y} \| > \sigma \| y \| \land t = kh, \ k \in \mathbb{N} \} \]

- Enforcing minimal inter-event time
  \[ t_{k+1} = \inf \{ t > t_k + T \mid \| y - \hat{y} \| > \sigma \| y \| \} \]

Output-feedback case

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PETC

- Periodic Event-Triggered Control (PETC)
  \[ t_{k+1} = \inf \{ t > t_k \mid \| y - \hat{y} \| > \sigma \| y \| \land t = kh, \ k \in \mathbb{N} \} \]

- Hybrid systems formulation
  \[
  \frac{d}{dt} \begin{bmatrix} \xi(t) \\ \tau \end{bmatrix} = \begin{bmatrix} A \xi + Bw \\ 1 \end{bmatrix}, \text{ when } \tau \in [0,h],
  \]
  \[
  \begin{bmatrix} \xi^+ \\ \tau^+ \end{bmatrix} = \begin{cases} J_1 \xi \\ 0 \end{cases}, \text{ when } \xi^\top Q \xi > 0, \ \tau = h
  \]
  \[
  \begin{cases} J_2 \xi \\ 0 \end{cases}, \text{ when } \xi^\top Q \xi \leq 0, \ \tau = h
  \]
  \[
  z = C \xi + Dw
  \]

---

Other references:

Hybrid systems formulation

\[
\frac{d}{dt} \begin{bmatrix} \xi \\ \tau \end{bmatrix} = \begin{bmatrix} A & Bw \\ 1 \end{bmatrix}, \quad \text{when } \tau \in [0, h],
\]

\[
\begin{bmatrix} \xi^+ \\ \tau^+ \end{bmatrix} = \begin{cases} J_1 \xi \\ J_2 \xi \\ 0 \end{cases}, \quad \text{when } \xi^T Q \xi > 0, \tau = h
\]

\[
z = C \xi + D w
\]

- In case \( w = 0 \) and interested in stability only
- Discretize at \( kh, k \in \mathbb{N} \) (just before jump) leading to discrete-time PWL system


\[\| z \|_{L_2} \leq \beta(\| \xi_0 \|) + \gamma_0 \| w \|_{L_2} \text{ with } \| z \|_{L_2} = \sqrt{\int_0^\infty \| z(t) \|^2 dt}\]
### Lifting-based approach

\[
\frac{d}{dt} \begin{bmatrix} \xi(t) \\ r_k(t) \end{bmatrix} = \begin{bmatrix} A\phi(\xi_k) + B_k w_k \\ C\phi(\xi_k) \end{bmatrix} \quad \text{when } \tau \in [0, h]
\]

\[
\begin{bmatrix} \xi(t) \\ r_k(t) \end{bmatrix}_+ = \begin{bmatrix} \phi(\xi) \\ 0 \end{bmatrix} \quad \text{when } \tau = h
\]

\[
z = C \xi + D w
\]

**Main result:** The hybrid system is internally stable and \(L_2\)-contractive iff the discrete-time nonlinear system is internally stable and \(J_2\)-contractive.

- \(L_2\)-contractive: there is \(\gamma_0 \in [0, 1)\) s.t.
  \[
  \|r\|_{\ell_2} \leq \beta(\|\xi_0\|) + \gamma_0 \|v\|_{\ell_2} \quad \text{with} \quad \|r\|_{\ell_2}^2 = \sum_{k=0}^{\infty} |r_k|^2
  \]

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### Time regularisation: Example

- **Periodic Event-Triggered Control (PETC)**
  \[
  t_{k+1} = \inf\{t > t_k \mid \|x^e(t) - x(t)\| > \sigma \|x(t)\| \wedge t = kh, \ k \in \mathbb{N}\}
  \]
  \[
  t_{k+1} = \inf\{t > t_k + T \mid C(x(t), e(t)) > 0\}
  \]
  \[
  t_{k+1} = \inf\{t > t_k + T \mid \eta^\prime(t) < 0\}
  \]
  \[
  \eta^\prime(t) = \psi(x, e, \eta)
  \]

---

### Dynamic event-triggered control

- **Static event generator:**
  \[
  t_{k+1} := \inf\{t > t_k + T \mid C(x(t), e(t)) > 0\}
  \]

- **How to find \(\psi\) and \(T\)?**

---

[1] Heemels, Dullerud, Teel, \(L_2\)-gain Analysis for a Class of Hybrid Systems with Applications to Reset and Event-triggered Control: A Lifting Approach


Recap: Design relative triggering

- Perturbation perspective:
  \[ \dot{x}(t) = Ax(t) + BK x(t_k) = (A + BK)x(t) + BKe(t) \]
- Since \( A + BK \) Hurwitz, quadratic Lyapunov function \( V(x) = x^\top P x \)
  \[ \frac{d}{dt} V \leq -a^2\|x(t)\|^2 + \|e(t)\|^2 \]
- Crux: Guarantee \( \|e(t)\| \leq \rho a \cdot \|x(t)\| \) with \( 0 < \rho < 1 \) s.t.
  \[ \frac{d}{dt} V \leq -a^2\|x(t)\|^2 + \|e(t)\|^2 \leq -(1 - \rho^2)a^2\|x(t)\|^2 \]
- Guarantee for Global Exponential Stability
  \[ t_{k+1} = \inf\{t > t_k \mid \|x(t) - x(t_k)\| \geq \rho a \cdot \|x(t_k)\| \} \]

Zeno-free: There is \( T > 0 \) such that \( t_{k+1} - t_k \geq T \) for all \( k \in \mathbb{N} \).

Dynamic event-triggered control

- Static event generator: \( t_{k+1} := \inf\{t > t_k + T \mid C(x(t), e(t)) > 0\} \)

Dynamic event generator [1,2,3]
  \[ \dot{\eta} = \Psi(x, e, \eta) \]
  \[ t_{k+1} := \inf\{t > t_k + T \mid \eta(t) < 0\} \]

- [1,2] design for \( w = 0 \) (no disturbances)
- Recently, [3] new design methodology for output-based decentralized triggering under disturbances (\( \mathcal{L}_2 \)-gain)

Basic design dETM

- Since \( A + BK \) Hurwitz, quadratic Lyapunov function \( V(x) = x^\top P x \)
  \[ \frac{d}{dt} V \leq -a^2\|x(t)\|^2 + \|e(t)\|^2 \]
- Now consider \( \dot{\eta} = \Psi(x, e, \eta) \)
  \[ \frac{d}{dt} U \leq -a^2\|x\|^2 + \|e\|^2 + \Psi \]
- To get \( \frac{d}{dt} U \leq -(1 - \rho^2)a^2\|x\|^2 - \epsilon\eta \) for some \( \epsilon > 0 \) we require
  \[ -a^2\|x\|^2 + \|e\|^2 + \Psi = -(1 - \rho^2)a^2\|x\|^2 - \epsilon\eta \]
  and thus \( \dot{\eta} = \Psi(x, e, \eta) = \rho^2 a^2\|x\|^2 - \epsilon\eta - \|e\|^2 \)
- Now \( t_{k+1} := \inf\{t > t_k + T \mid \eta(t) < 0\} \), \( \eta(0) = 0 \) and \( t_0 = 0 \):
  - \( -\eta(t) \geq 0 \) for \( t \in \mathbb{R}_{\geq 0} \) and thus \( U \) positive definite
  - \( -\frac{d}{dt} U \leq -(1 - \rho^2)a^2\|x\|^2 - \epsilon\eta \) and thus GES
- Never triggers before the static version!!

Dynamic ETC: Example

Case study: \( \mathcal{L}_2 \)-gain \( \theta = 4 \) from input \( w \) to state \( x \): \( \tau_{mct} = 9.1 \cdot 10^{-3} \)

- Dynamic event generator \( t_{k+1} := \inf\{t > t_k + \tau_{mct} \mid \eta(t) < 0\} \)
- Static event generator: \( t_{k+1} := \inf\{t > t_k + \tau_{mct} \mid \Psi(x, e, \tau, \eta) < 0\} \)

---

Dynamic ETC: Example

Case study: $L_2$-gain $\theta = 4$ from input $w$ to state $x$: $\tau_{\text{miet}} = 9.1 \cdot 10^{-3}$

- **Dynamic event generator** $t_{k+1} := \inf \{ t > t_k + \tau_{\text{miet}} \mid \eta(t) < 0 \}$
- **Static event generator**: $t_{k+1} := \inf \{ t > t_k + \tau_{\text{miet}} \mid \Psi(x,e,\tau,\eta) < 0 \}$

Motivation

Cooperative Adaptive Cruise Control

- String stability: disturbance attenuation along the vehicle string
  - $L_p$-gain $\leq 1$
- Communication resources limited $\rightarrow$ event-triggered communication
Conclusions

- Event-triggered control: A new resource-aware control paradigm
- Several ETC algorithms discussed with their own tools (hybrid)
- Challenges
  - Performance / Robustness w.r.t. disturbances
  - Output-based & decentralized event generators
  - Constrained systems (MPC)
  - Implementation and Applications
  - Better than periodic time-triggered control
  - Improved analysis and design tools: MIET, average inter-execution times, $L_\infty$-gain, etc.
- Many interesting practical and theoretical issues open in this appealing research field

More info: http://www.heemels.tue.nl
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Literature


Recent overviews:

Pointers for “better than periodic time-triggered control:"
- Astrom, Bernhardsson Comparison of periodic and event based sampling for first order stochastic systems, IFAC World Congress 1999

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